The role of multiphase instabilities in Nature's extremes

Jenny Suckale

SIGMA group, Stanford University

SIGMA

SImulations of Geophysical Multi-phAse flows


ICE

FIRE

ROCK

WATER

Fracture or flow?

FRACTURE
but:
Aseismic deformation and creep?

FLOW
but:
Eruption speeds of 100s m/s?

Fracture ~~or~~ and? flow?

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but:
Aseismic deformation and creep?

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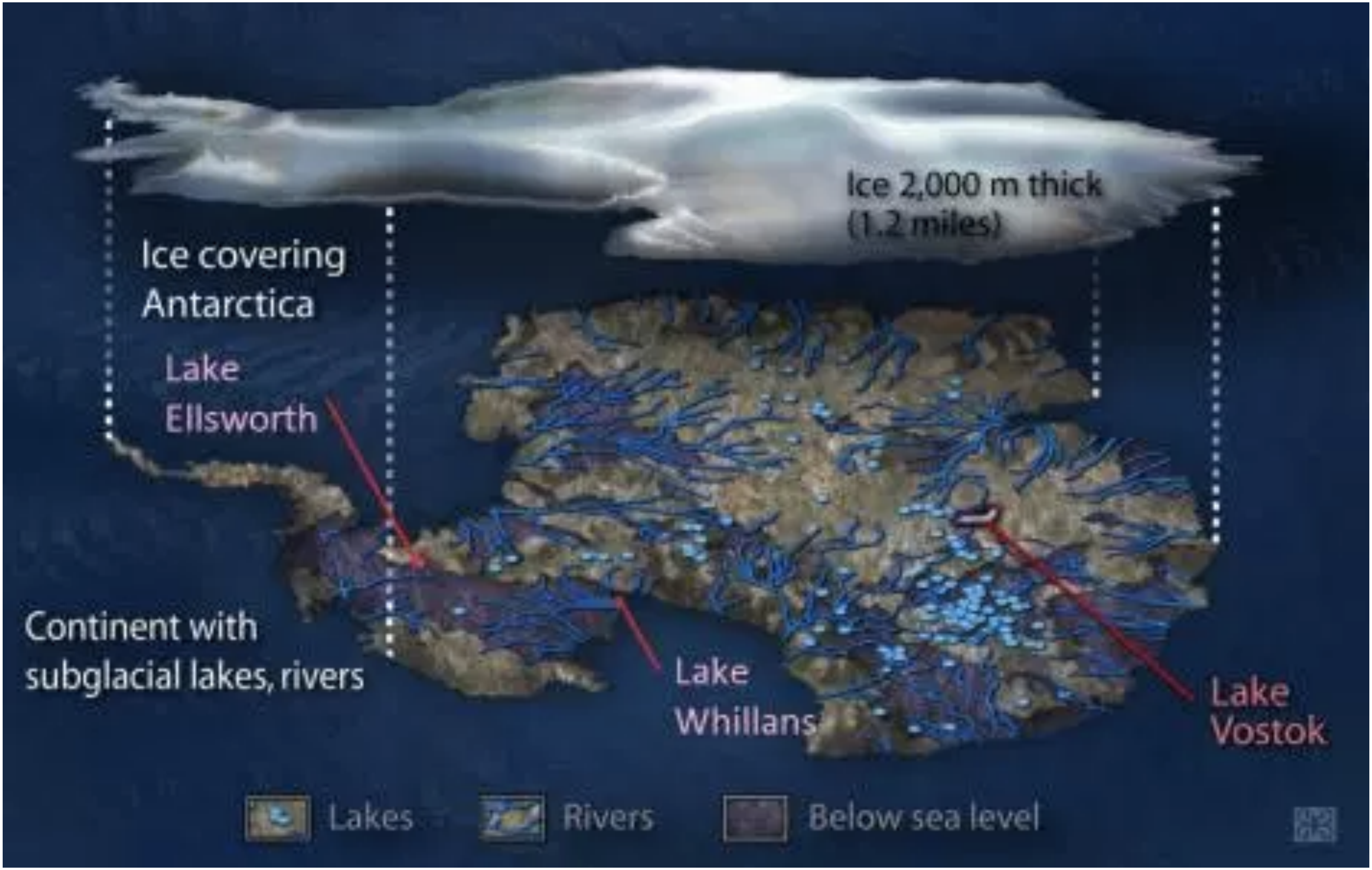
Fracture ~~or~~ and? flow?

FRACTURE
Sliding over a hydrologically flushed fault zone

FLOW
Stability of a solid matrix experiencing sustained flow

Fracture ~~or~~ and? flow?

FRACTURE
Sliding over a hydrologically flushed fault zone



Artistic representation by Zina Deretsky, National Science Foundation

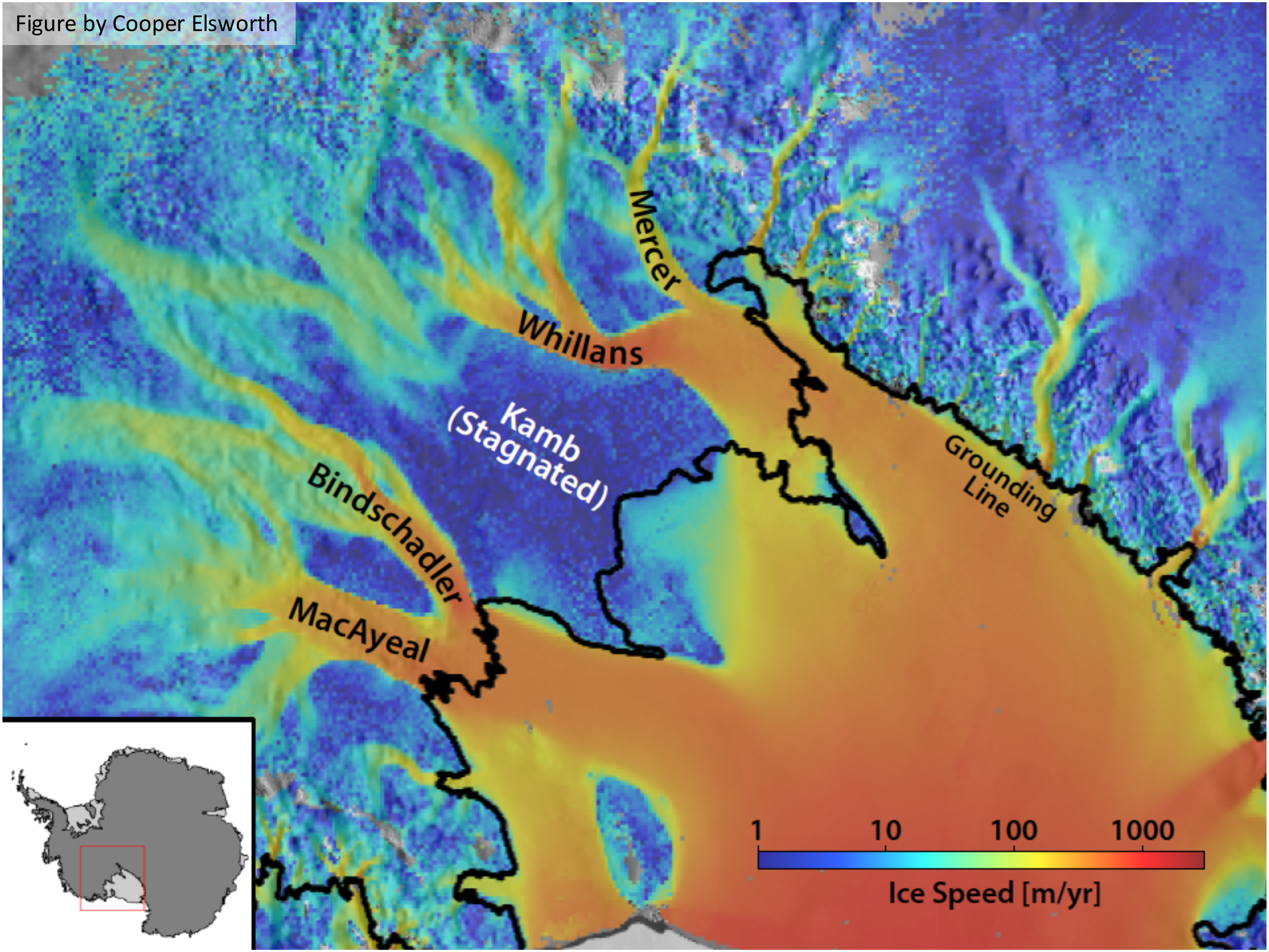


Figure by Cooper Elsworth
Mercer
Whillans
Kamb
(Stagnated)
Bindschadler
MacAyeal
Grounding
Line
1
10
100
1000
Ice Speed [m/yr]

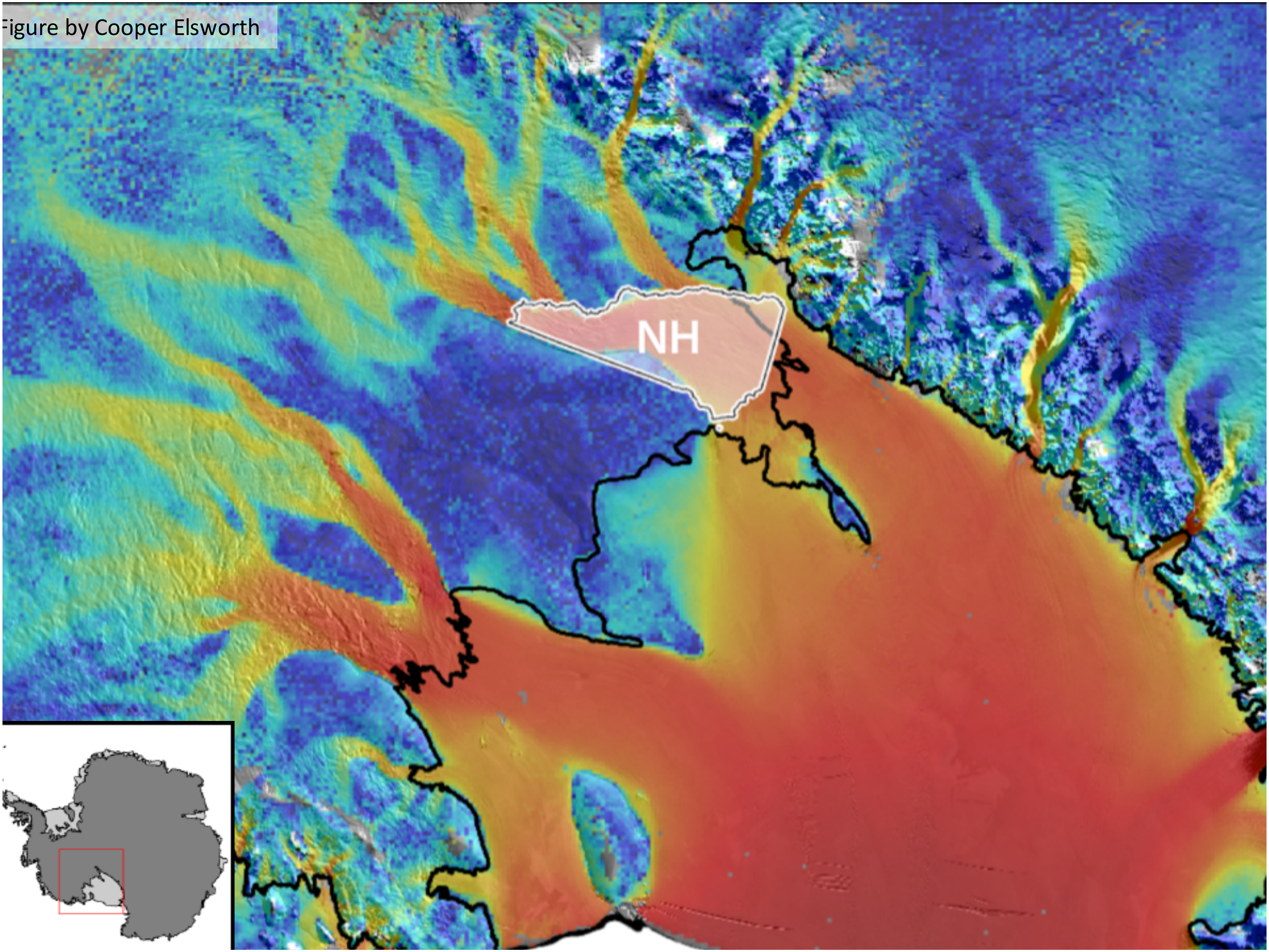


Figure by Cooper Elsworth
NH

Photo courtesy of Kurt Cuffey, Berkeley

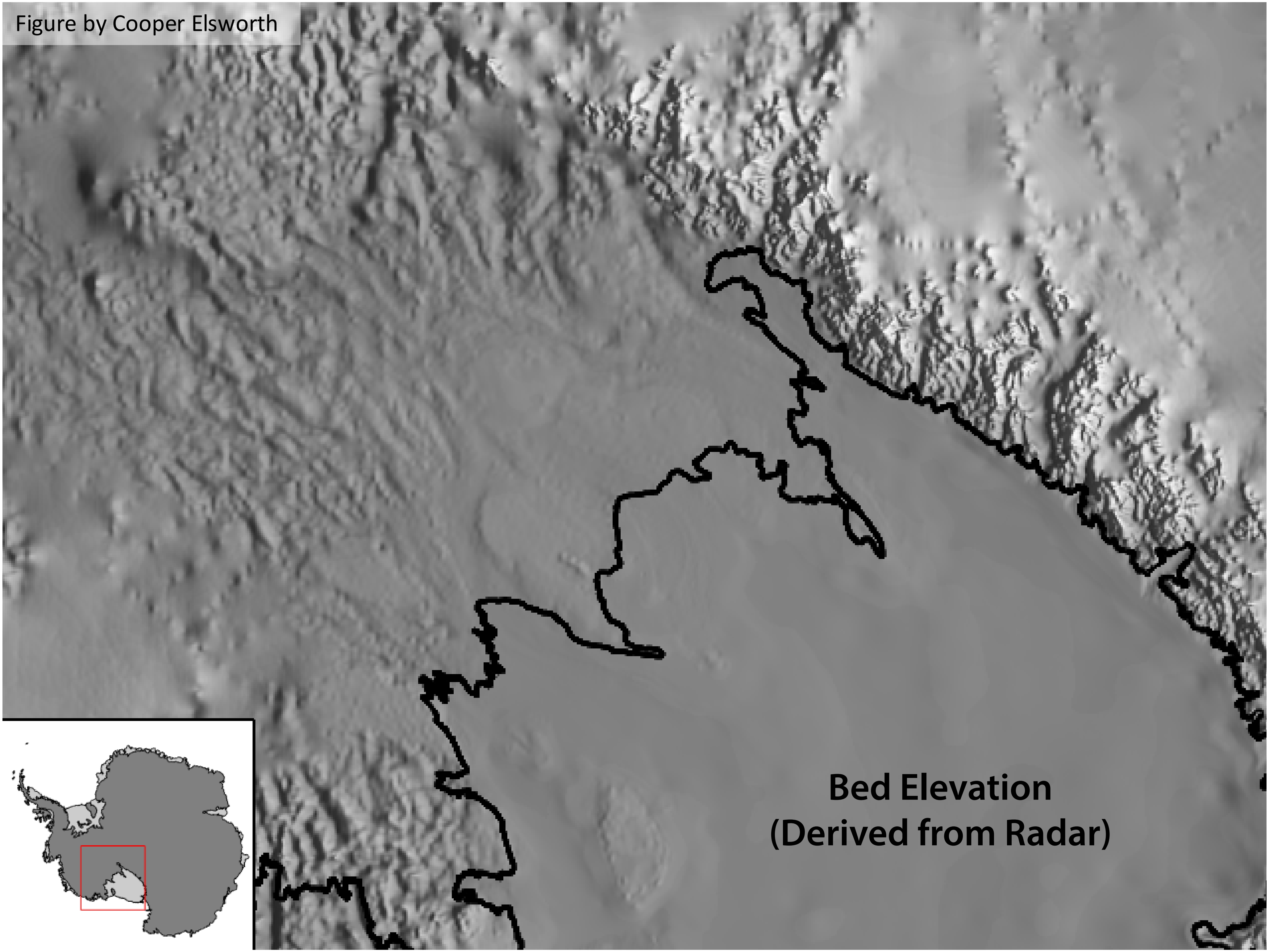


Figure by Cooper Elsworth
Bed Elevation
(Derived from Radar)

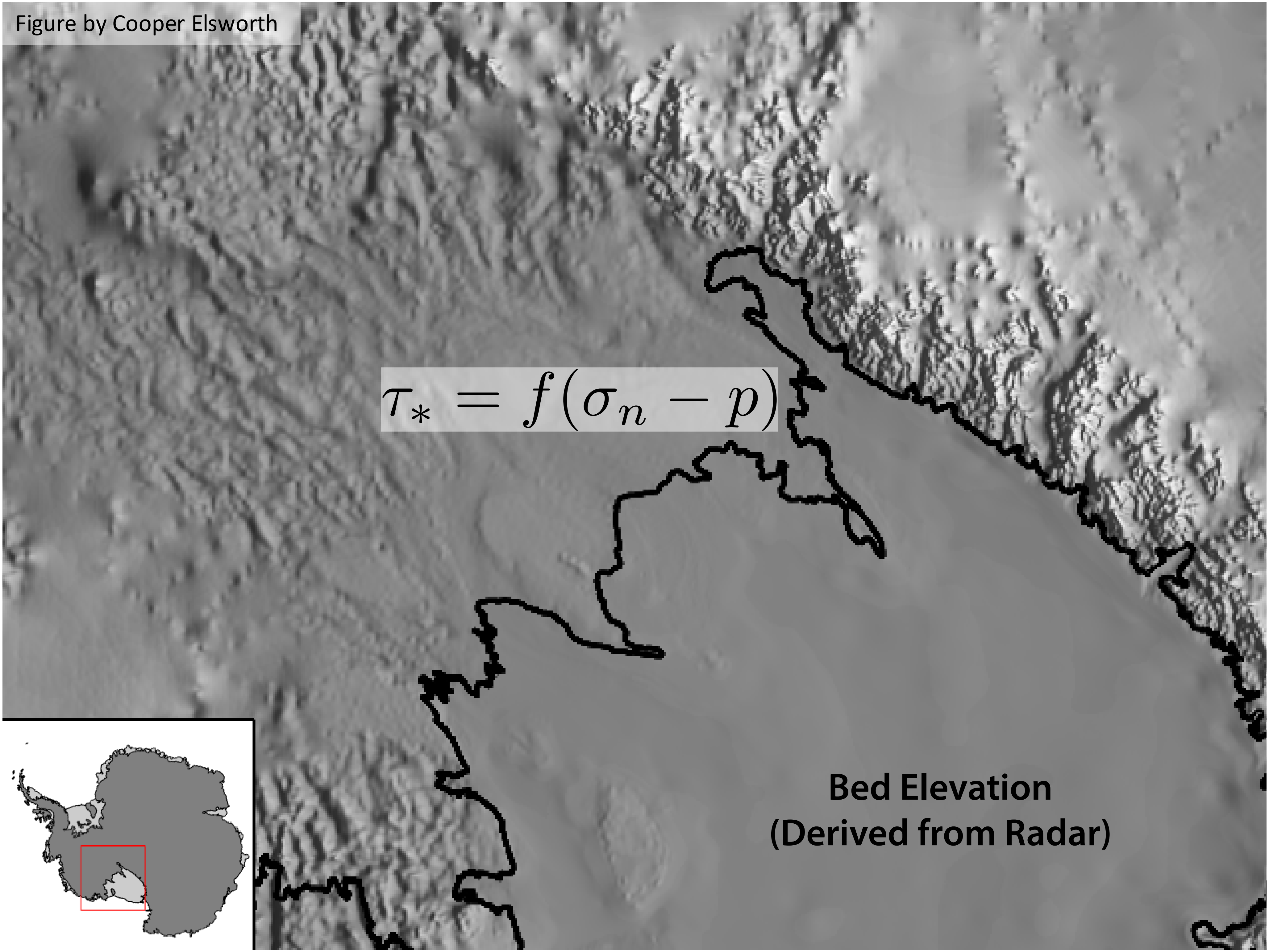


Figure by Cooper Elsworth
$\tau_* = f(\sigma_n - p)$
Bed Elevation
(Derived from Radar)

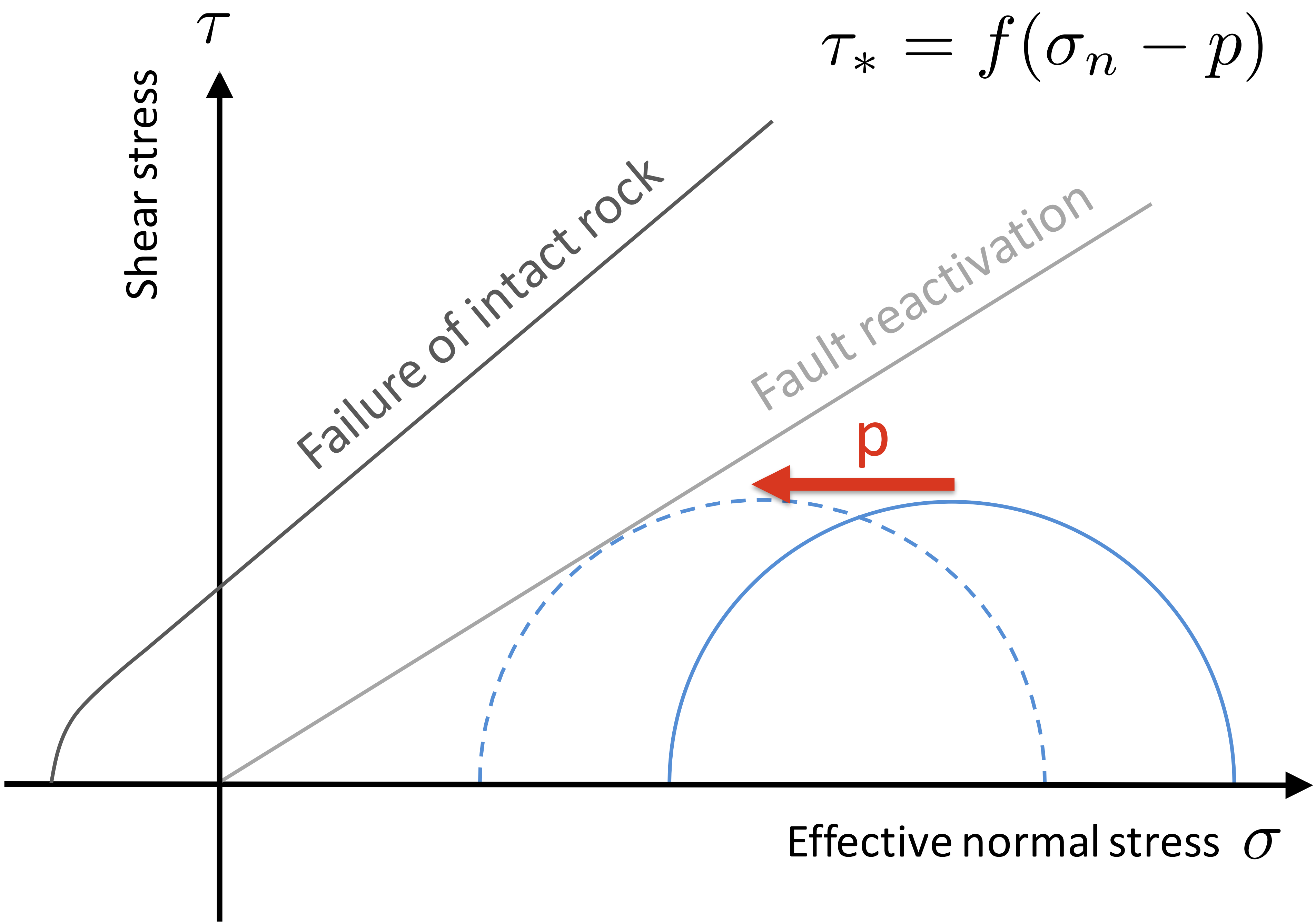
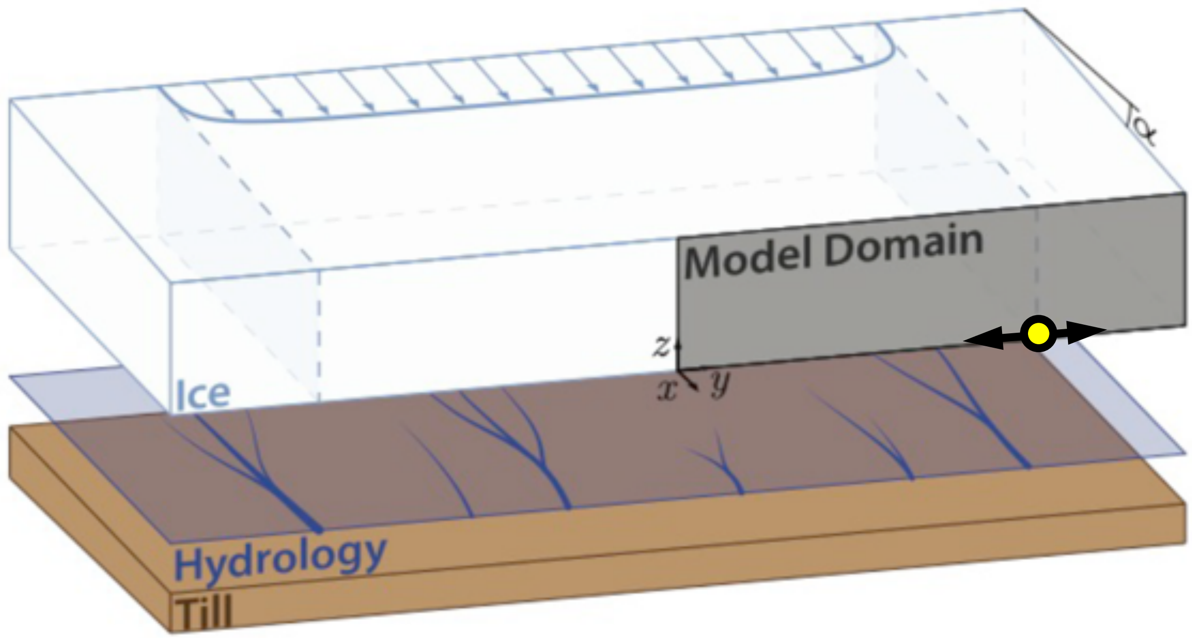
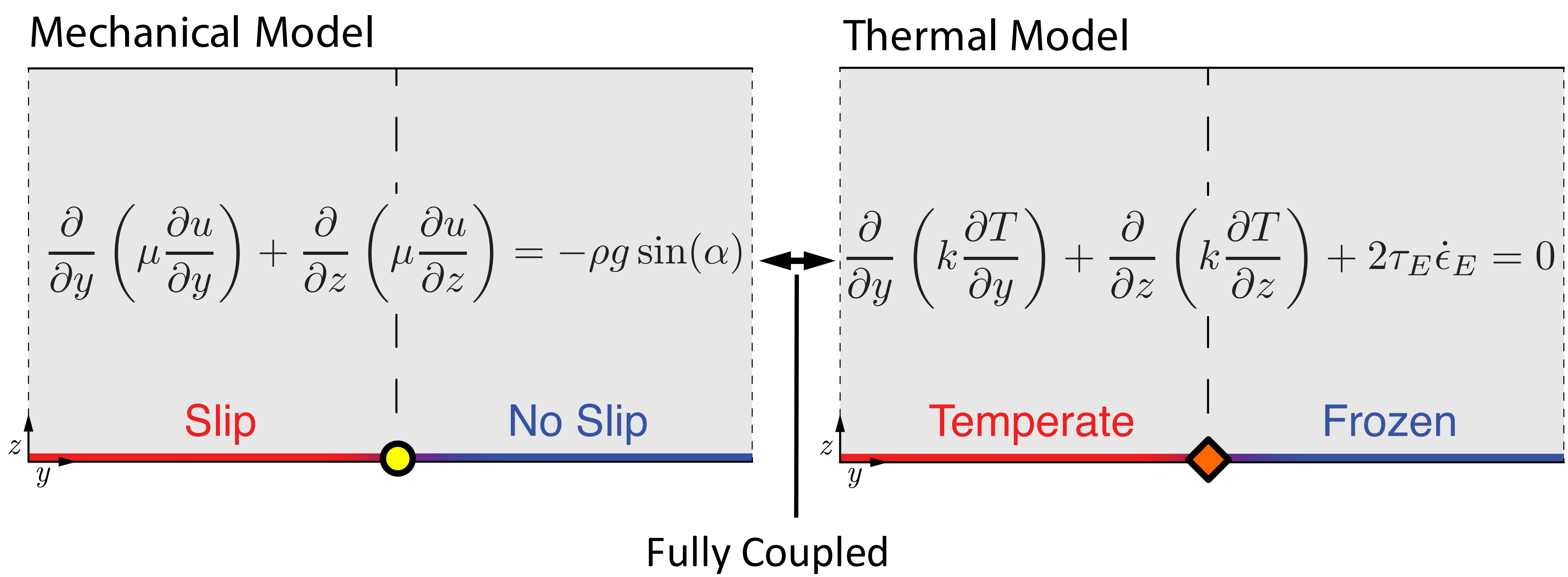

τ
$\tau_* = f(\sigma_n - p)$
Shear stress
Failure of intact rock
Fault reactivation
p
Effective normal stress σ

Photo courtesy of Mike Hambrey, Taylor Glacier, Antarctica, 1987



Cooper Elsworth



Generalized model by Cooper Elsworth; see Elsworth and Suckale, 2016

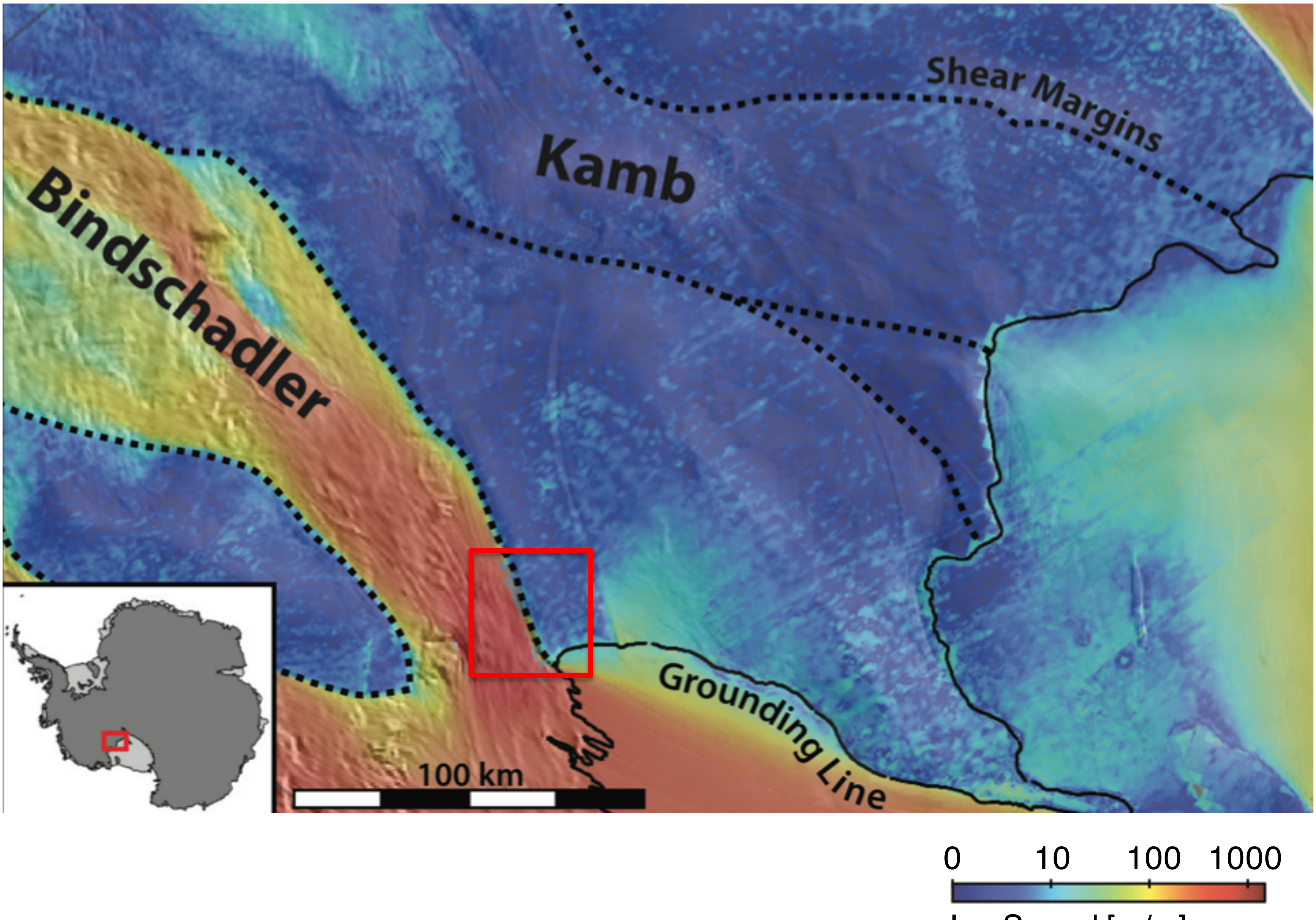
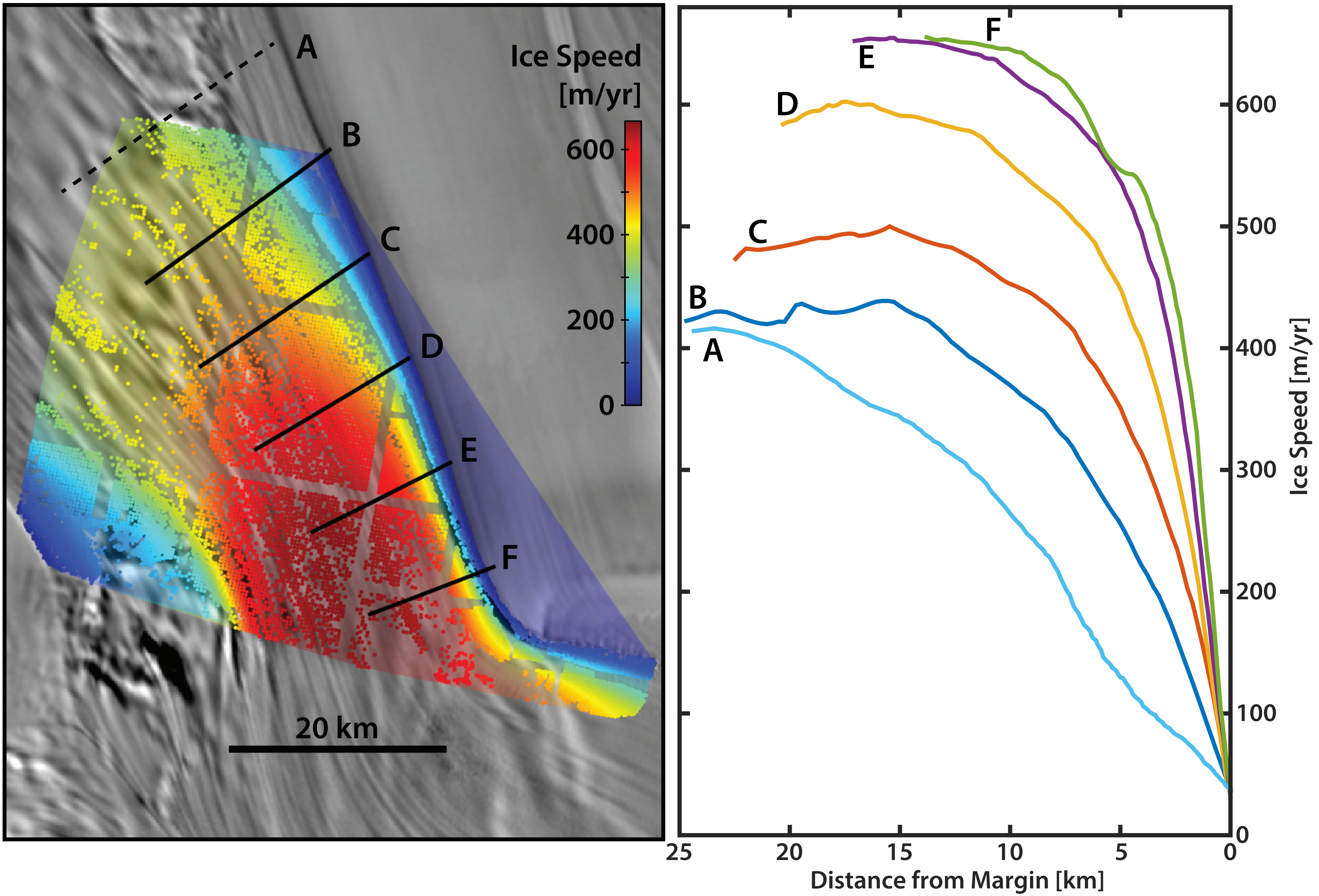
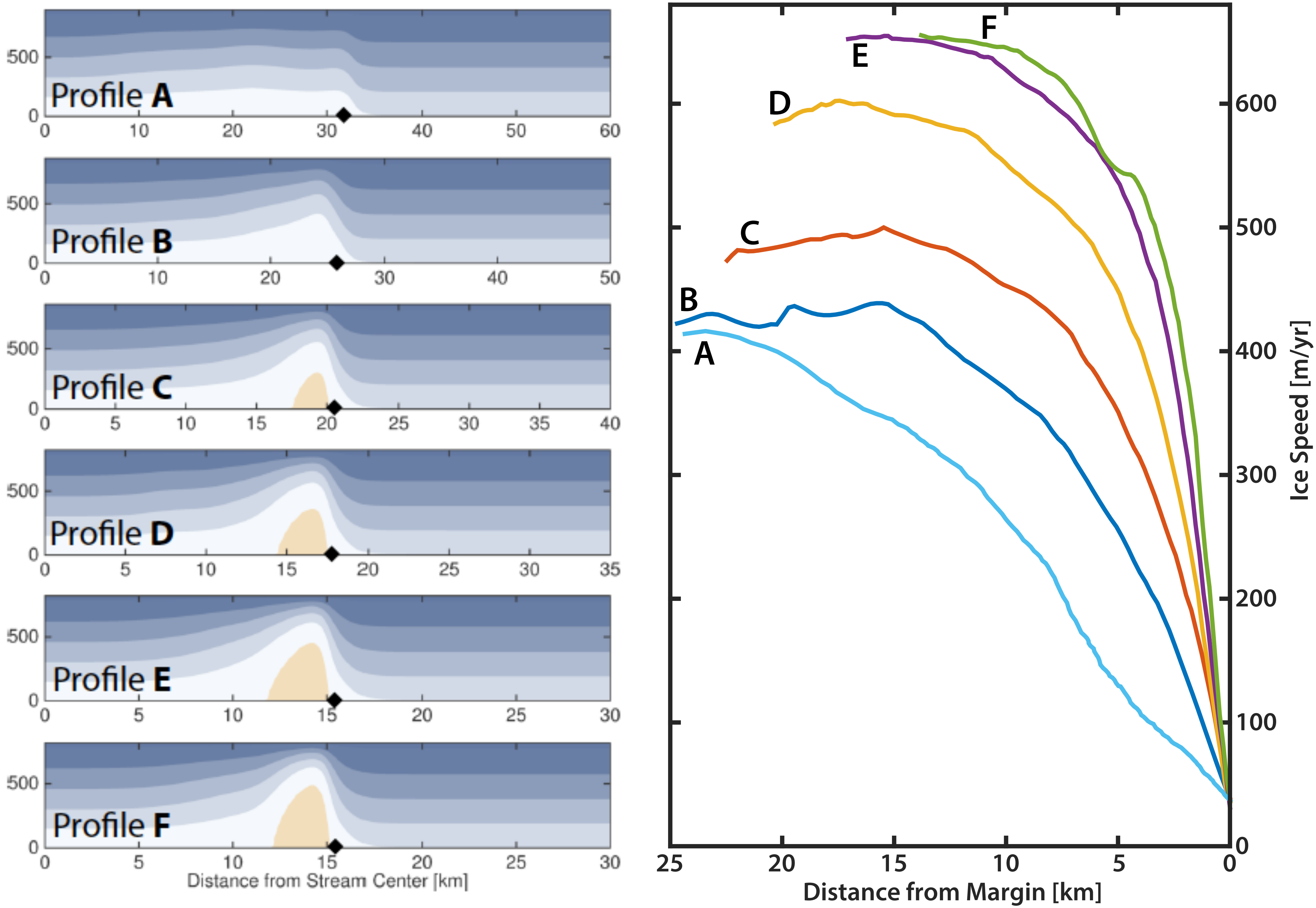


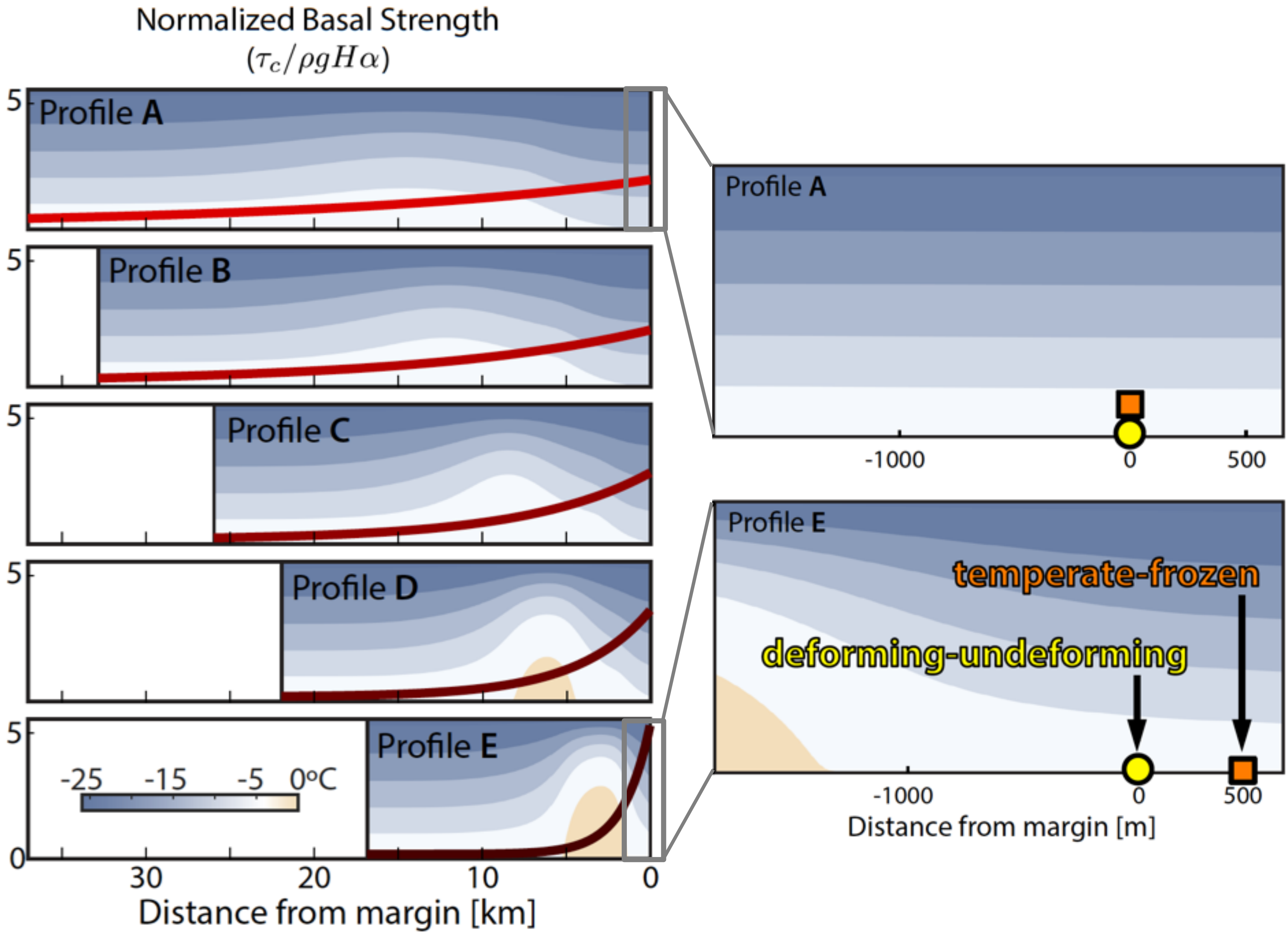
Figure by Cooper Elsworth



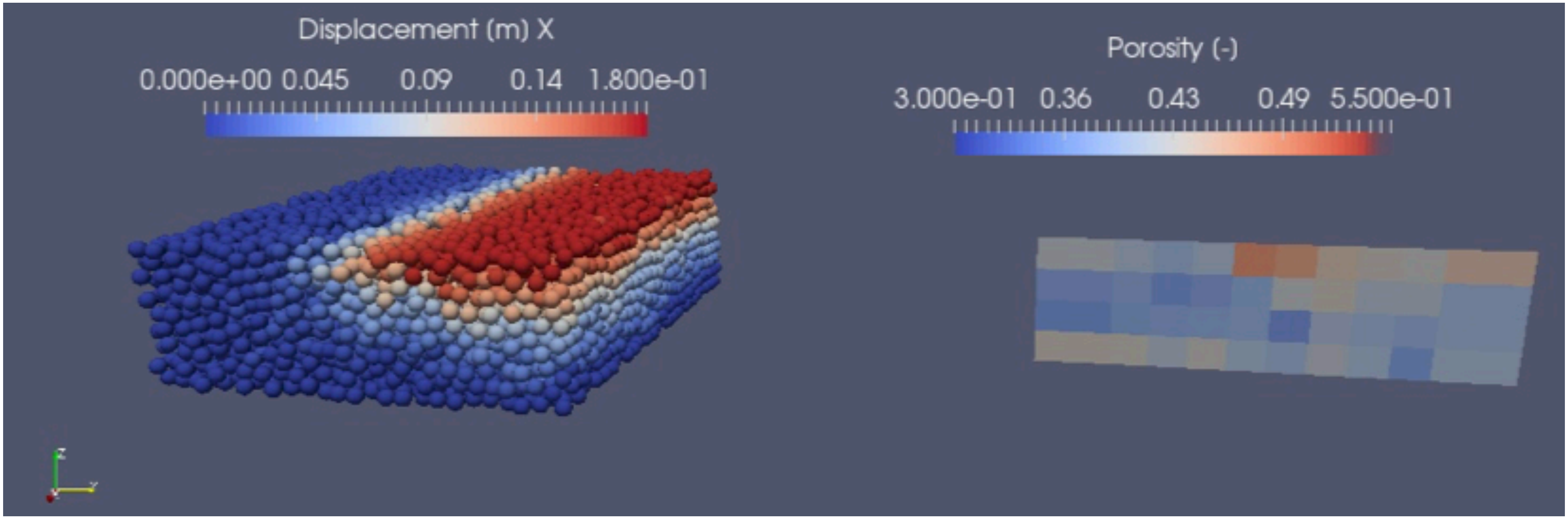
Data from NSIDC, Scambos et al., 1994



Simulations by Cooper Elsworth; see Elsworth and Suckale, 2016



Elsworth and Suckale, 2016



Grains:

$$\frac{\partial^2 \boldsymbol{x}^i}{\partial t^2} m^i = \boldsymbol{f}_{\mathrm{g}}^i + \sum_{j \in N} \left(\boldsymbol{f}_{\mathrm{n}}^{i,j} + \boldsymbol{f}_{\mathrm{t}}^{i,j} \right) + \boldsymbol{f}_{\mathrm{f}}^i$$

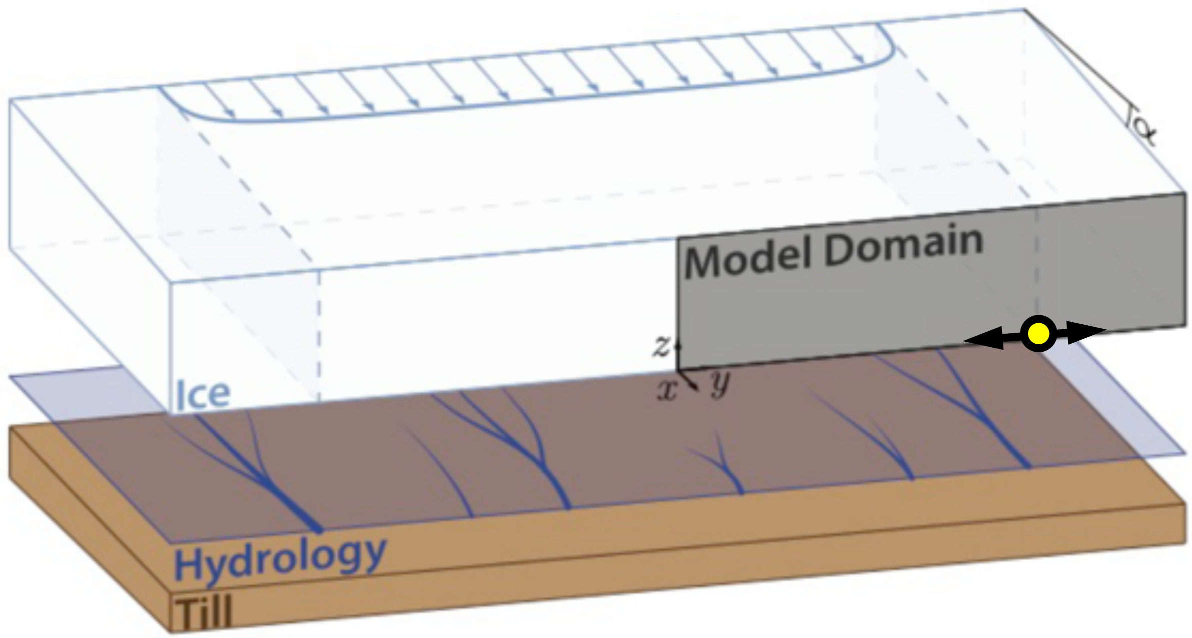
Water:

$$\frac{\partial P}{\partial t} = \frac{1}{\beta \Phi \eta} \nabla \cdot [k \nabla P] - \frac{1}{\beta \Phi (1 - \Phi)} \frac{\partial \Phi}{\partial t}$$

Indraneel Kasmalkar

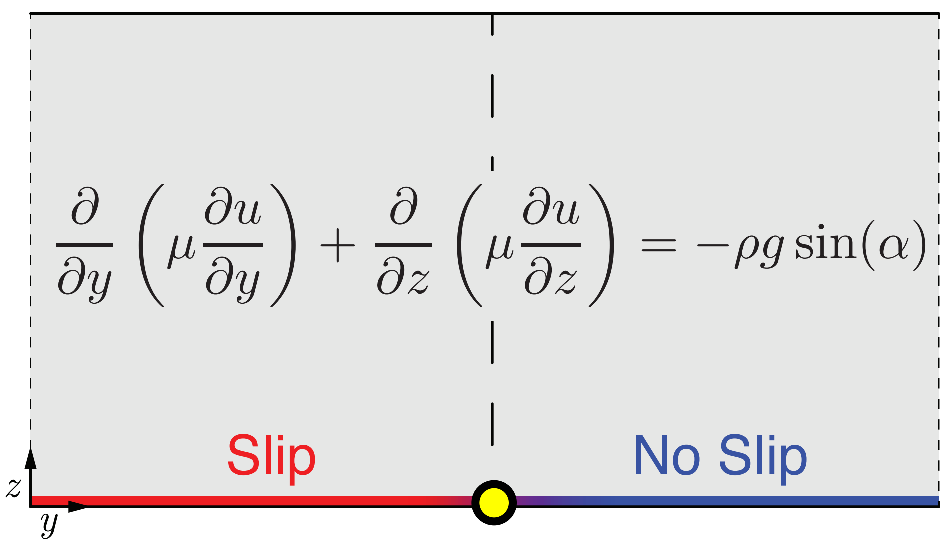
→ Flow channelization occurs first at the shear margin

Simulations by Indraneel Kasmalkar; Collaboration with Anders Damsgaard and Liran Goren



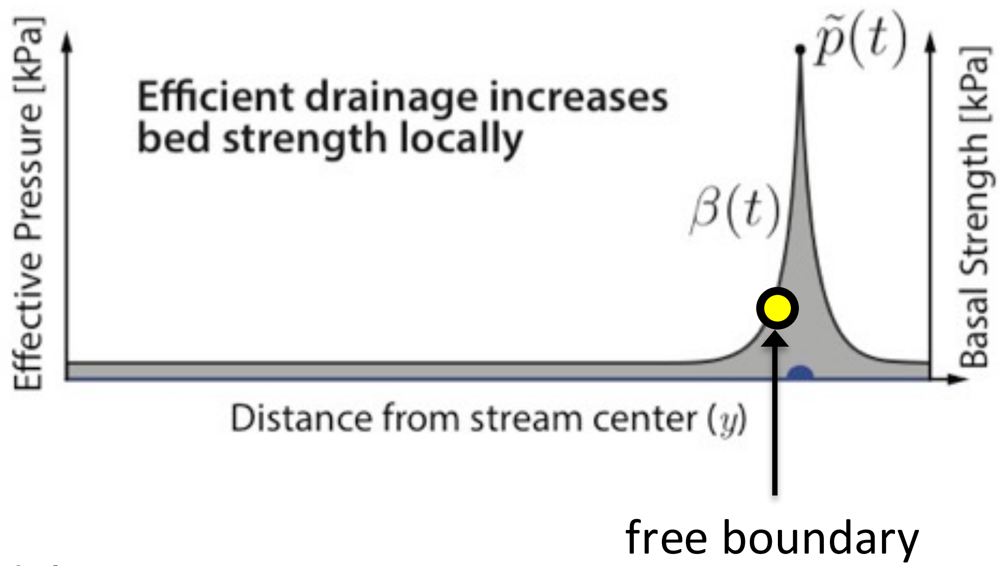
Cooper Elsworth

Mechanical Model

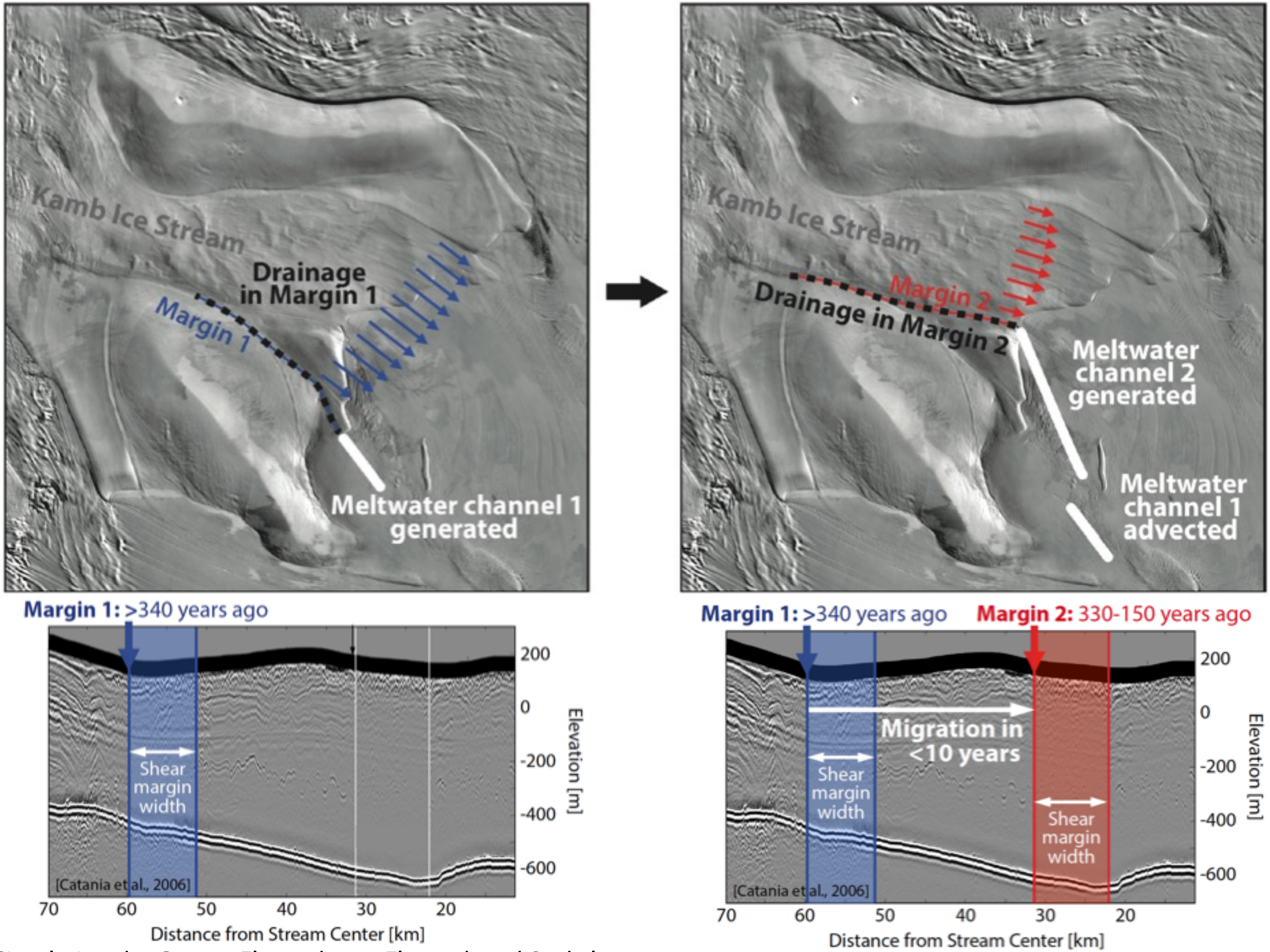


$$\frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu \frac{\partial u}{\partial z}\right) = -\rho g \sin(\alpha)$$

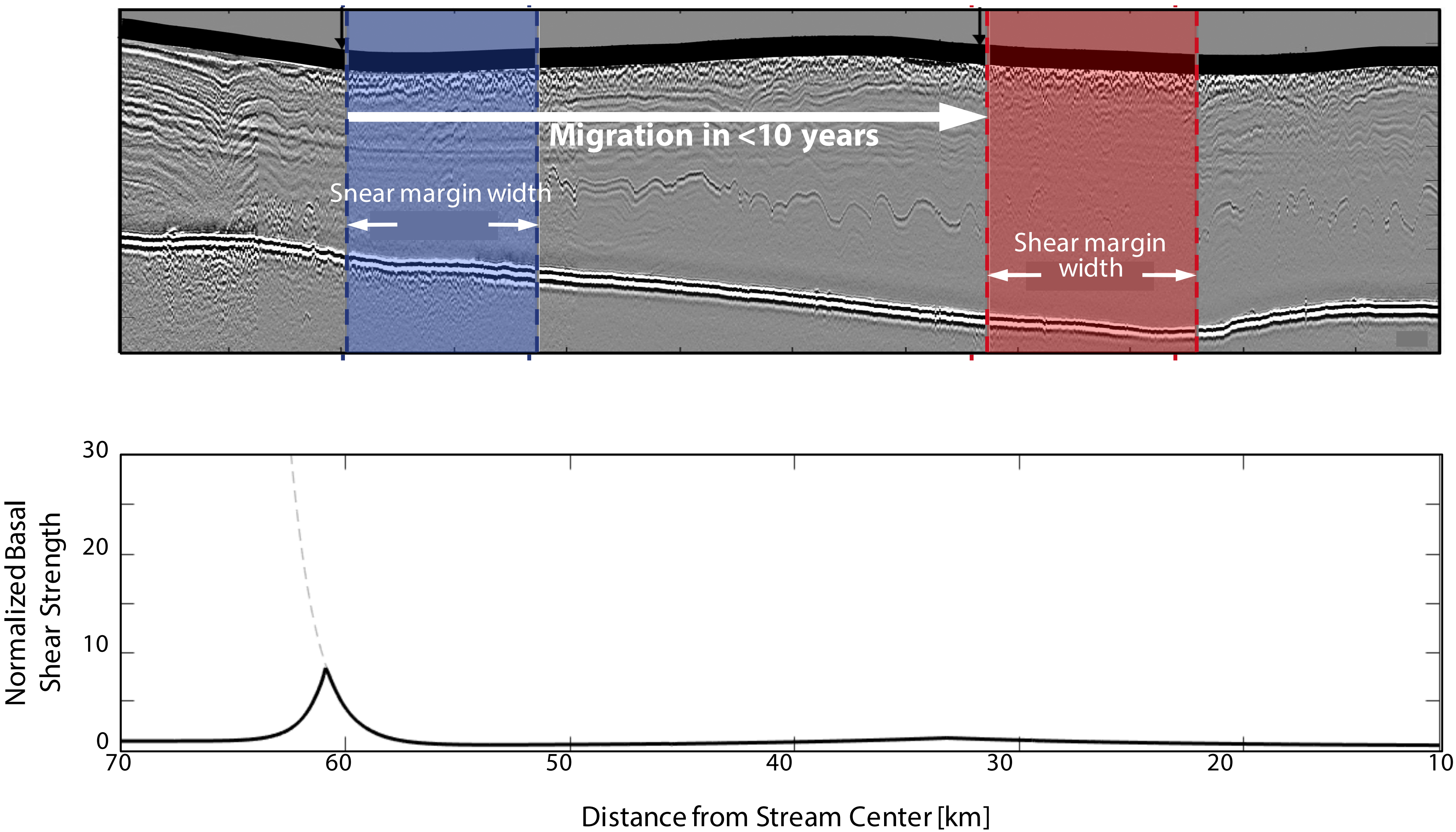
$$\tau_* = f(\sigma_n - p)$$



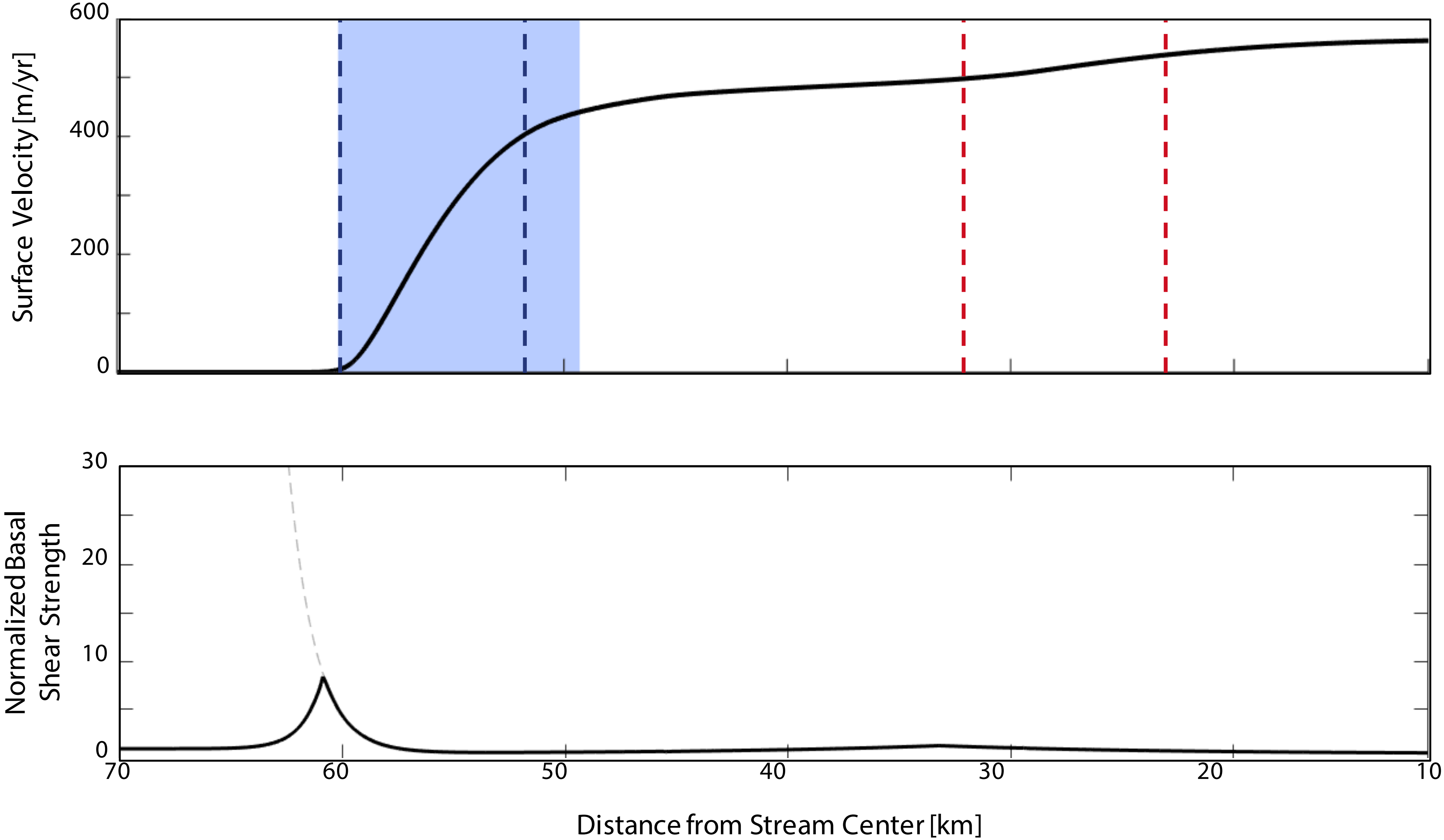
Generalized model by Cooper Elsworth; see Elsworth and Suckale, 2016



Simulations by Cooper Elsworth; see Elsworth and Suckale, 2016



Simulations by Cooper Elsworth; see Elsworth and Suckale, 2016



Simulations by Cooper Elsworth; see Elsworth and Suckale, 2016

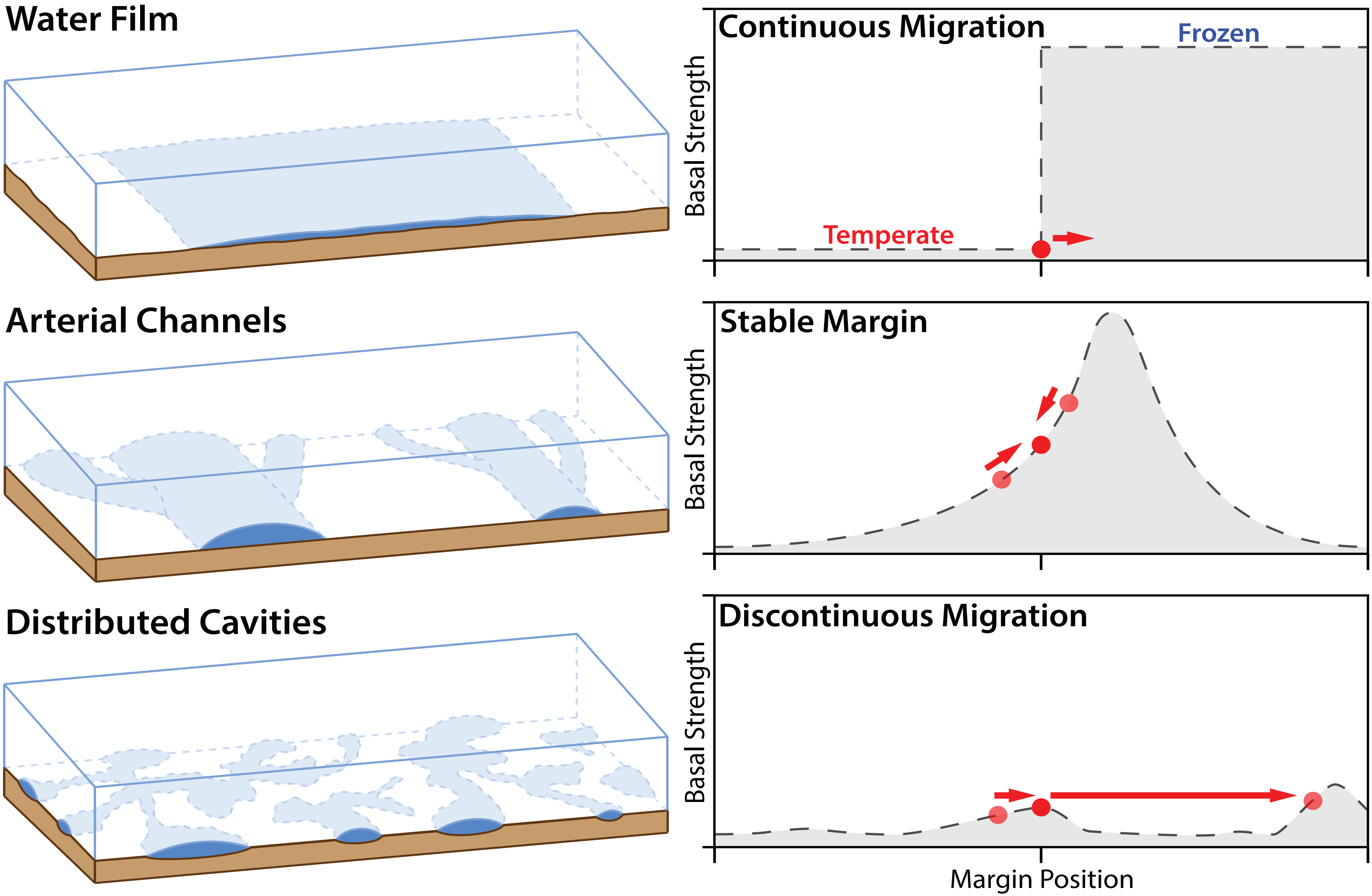
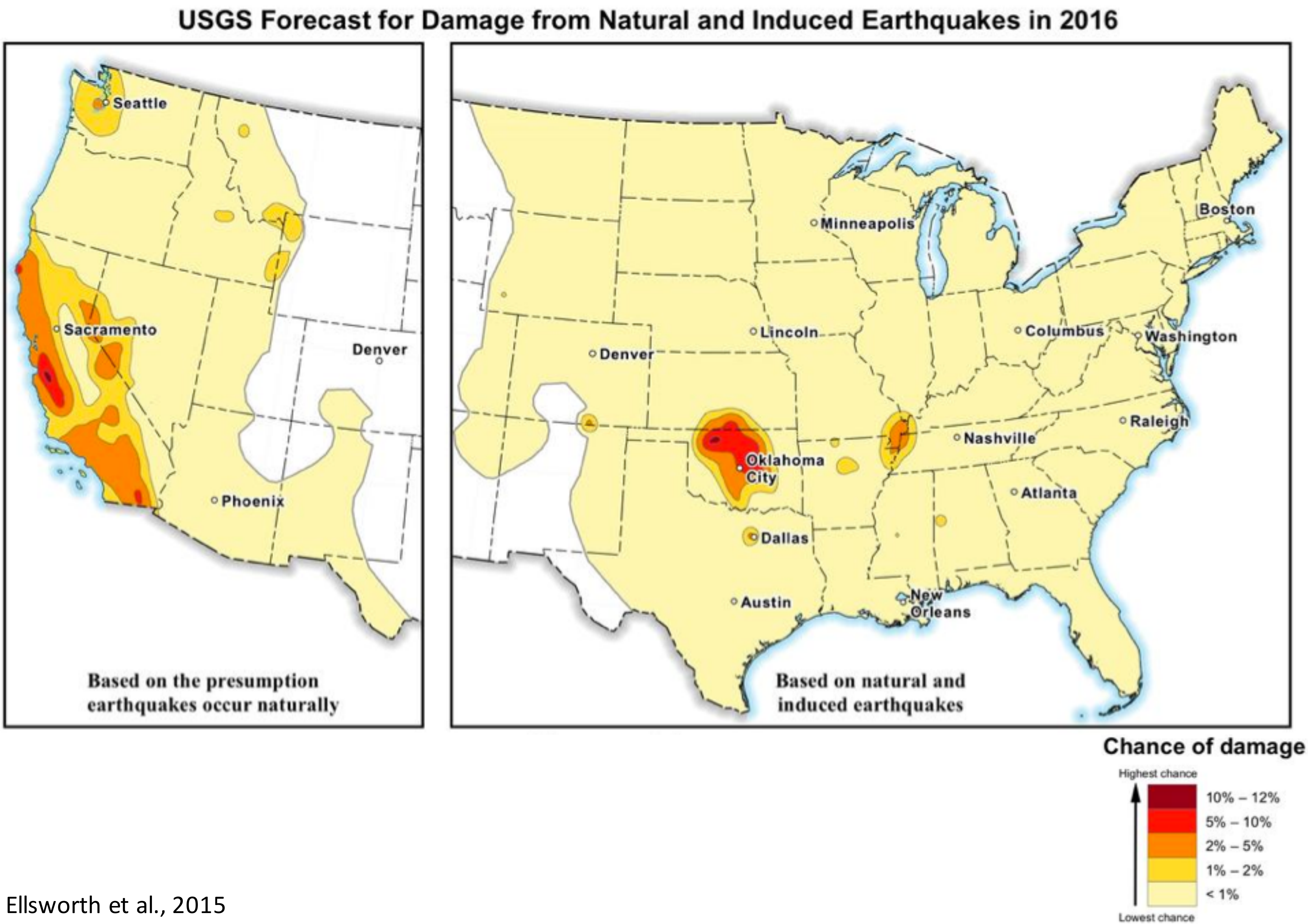


Figure by Cooper Elsworth

Fracture ~~or~~ and? flow?

FRACTURE
Sliding over a hydrologically flushed fault zone

- Flow creates a patchy fault surface (here: dry patches act as asperities, lakes are zones of free slip)
- Flow introduces a different time scale that can lead to rapid rearrangement of the slipping zone.



Ellsworth et al., 2015

Injection rate in million of barrels (≈10^8 liter) per month

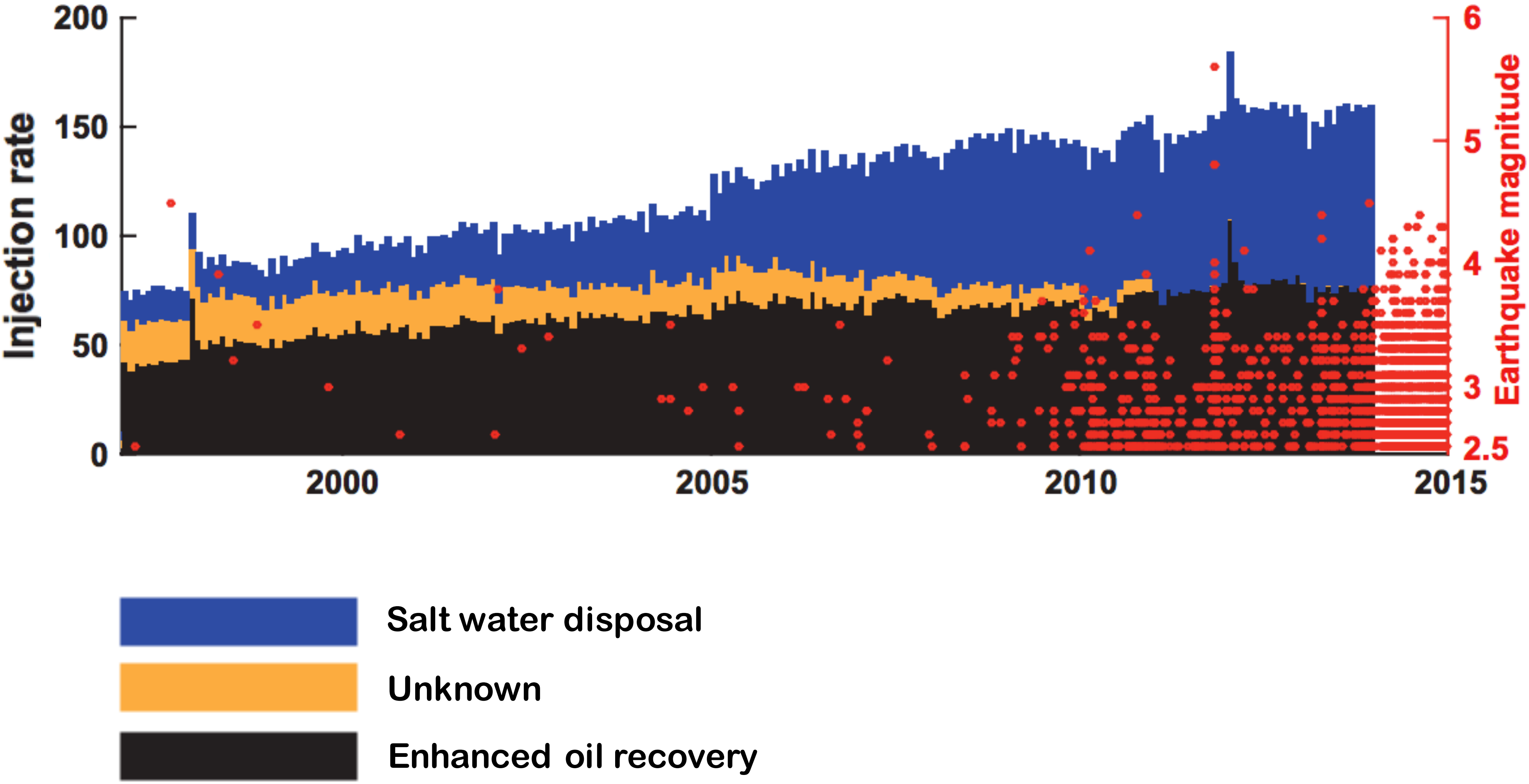
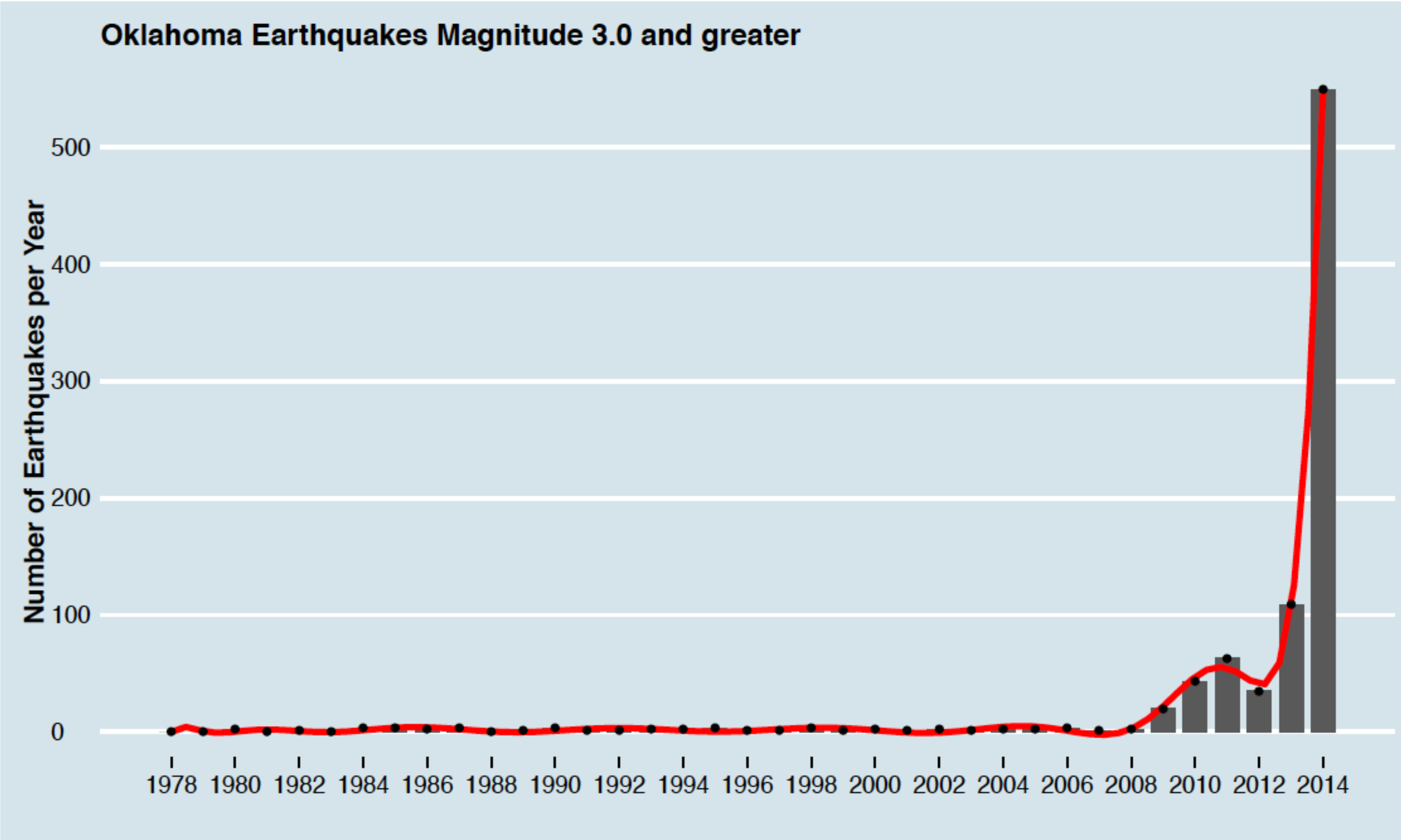


Figure modified from Walsh and Zoback, 2016



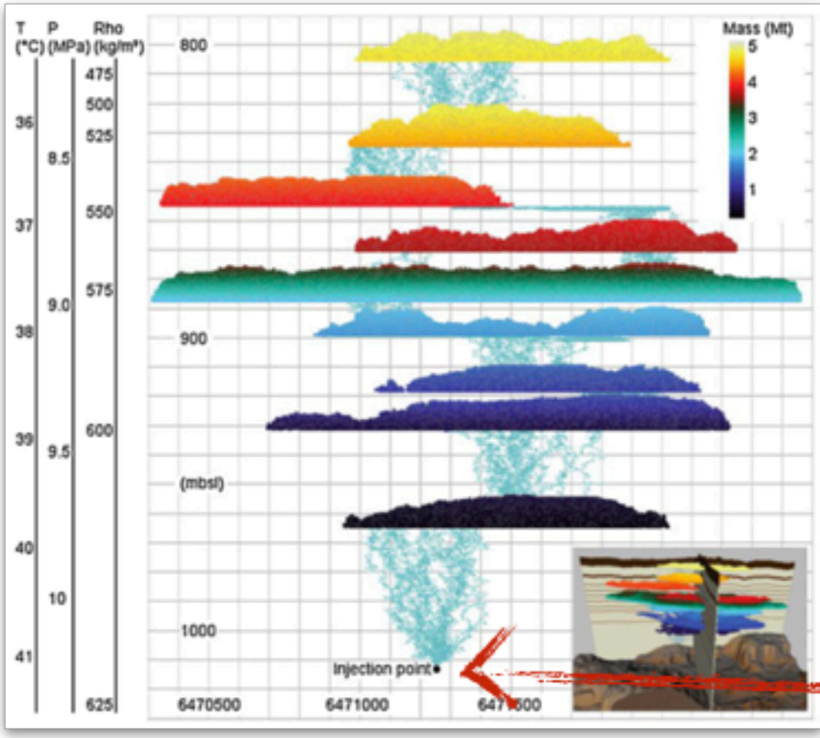
Oklahoma Earthquakes Magnitude 3.0 and greater
Number of Earthquakes per Year
0
100
200
300
400
500
1978
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014

Fracture ~~or~~ and? flow?

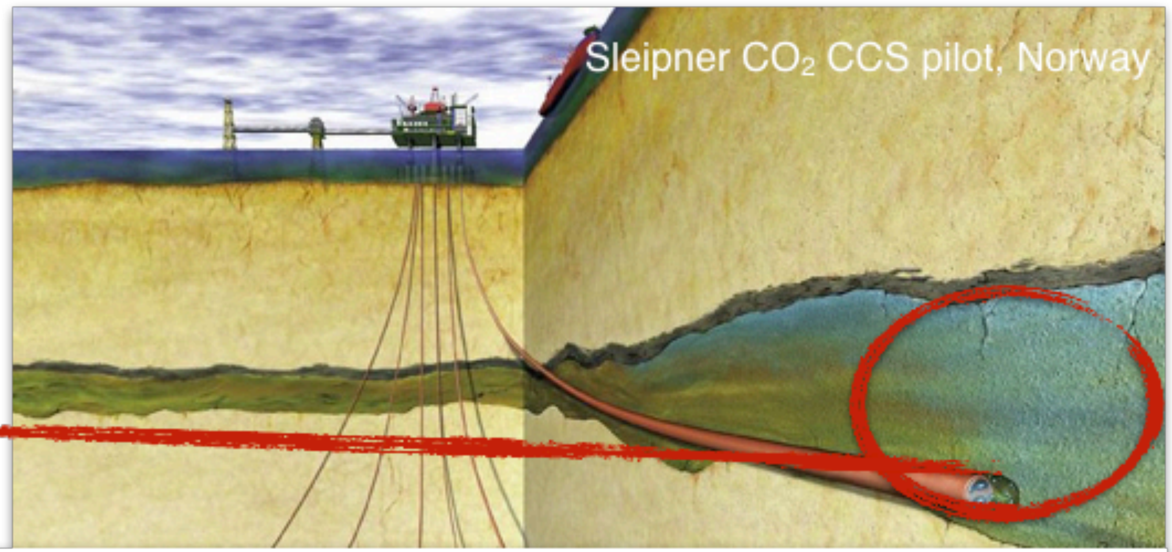
FLOW

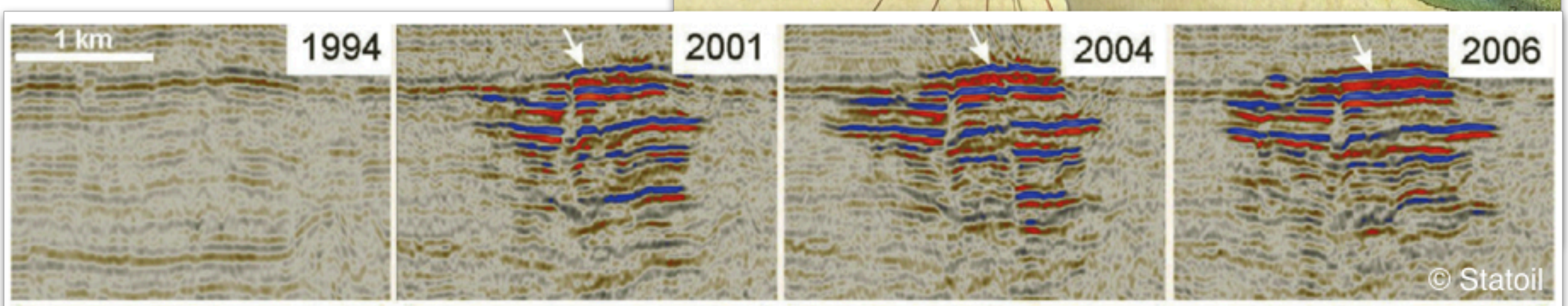
Stability of a solid matrix experiencing sustained flow

Sleipner gas field - the world's first offshore carbon capture and storage plant



Cavanagh and Haszeldine, 2014





Chadwick et al., 2009

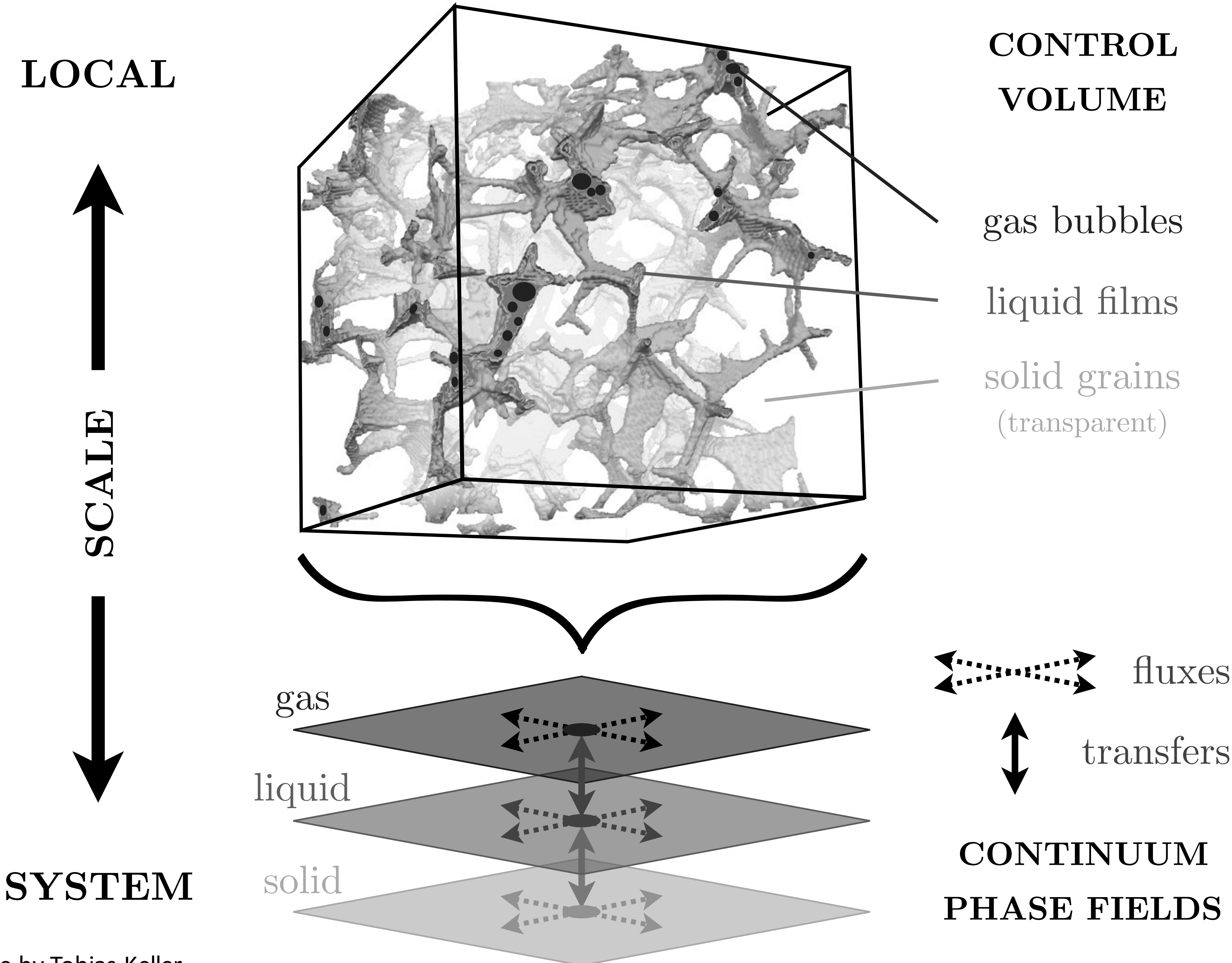


Figure by Tobias Keller

RATE OF CHANGE | DIFFUSIVE FLUX | EXT SOURCE

$$\frac{\partial^i \phi^i \rho^i a^i}{\partial t} + \nabla\cdot\left(a^i \rho^i \mathbf{q}^i_\phi\right) + \nabla\cdot\mathbf{q}^i_a + \Gamma^i_a + Q^i_a = \Upsilon^i_a$$

MASS FLUX | PHASE TRANSFER | INT PRODUCTION


Tobias Keller

DIFFUSIVE FLUXES	$\mathbf{q}^i_\phi$ volume flux	$\mathbf{q}^i_s$ entropy flux
	$\underline{\mathbf{q}}^i_v$ momentum flux (stress)	$\mathbf{q}^i_j$ chemical flux
PHASE TRANSFERS	Γ^i_ϕ volume transfer	Γ^i_s entropy transfer
	$\vec{\Gamma}^i_v$ momentum transfer	Γ^i_j chemical transfer
EXT SOURCE	Q^i_ϕ none	Q^i_s radiogenic heating
	$\mathbf{Q}^i_v$ gravity body force	Q^i_j none
INT PRODUCTION	Υ^i_ϕ none	Υ^i_s entropy production
	Υ^i_v none	Υ^i_j none

Tobias Keller

Multiphase mechanics

Tobias Keller

segregation velocity | segregation coefficient | pressure gradients | viscous stress | phase buoyancy

SEGREGATION $$\mathbf{v}_\Delta^i = \phi^i \Delta \mathbf{v}^{i*} = -\frac{q_\phi^i}{\eta_\phi^i}\left(\phi^i \nabla P^* + \nabla P_\Delta^i - \nabla \cdot \phi^i \eta_\phi^i \mathbf{D}\left(\mathbf{v}^i\right) - \phi^i \rho^i \mathbf{g}\right)$$

COMPACTION $$P_\Delta^i = \phi^i \Delta P^{i*} = -\frac{\eta_\phi^i}{r_\phi^i}\left(\phi^i \nabla \cdot \mathbf{v}^* + \nabla \cdot \mathbf{v}_\Delta^i - \frac{\phi^i}{\rho^i}\frac{D^i \rho^i}{Dt} - \frac{\Gamma^i}{\rho^i}\right)$$

compaction pressure | compaction coefficient | velocity divergences | phase compressibility | mass transfer

Keller and Suckale, in review

Multiphase mechanics

Tobias Keller

segregation velocity | segregation coefficient | pressure gradients | viscous stress | phase buoyancy

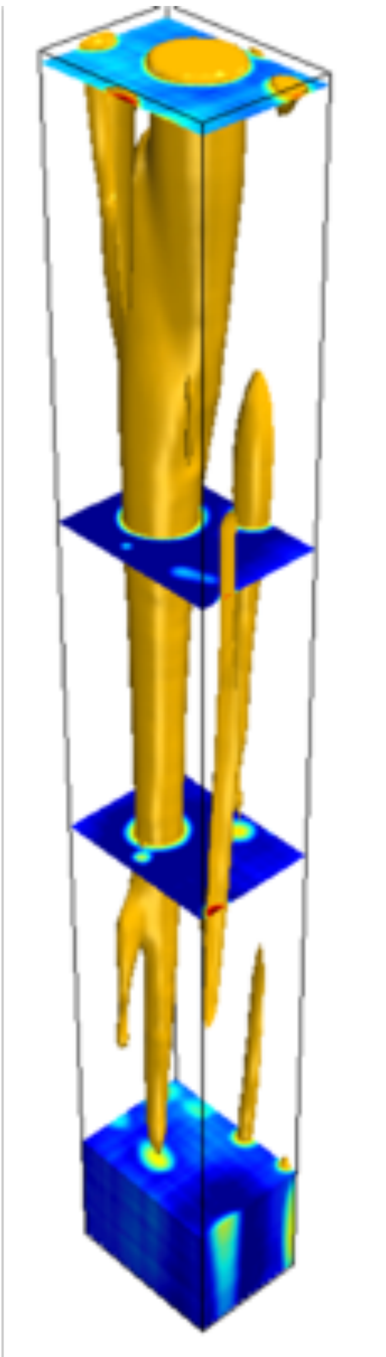
SEGREGATION
$$\mathbf{v}^i_\Delta = \phi^i \Delta \mathbf{v}^{i*} = -\frac{q^i_\phi}{\eta^i_\phi}\left(\phi^i \nabla P^* + \nabla P^i_\Delta - \nabla\cdot\phi^i\eta^i_\phi \mathbf{D}\left(\mathbf{v}^i\right) - \phi^i\rho^i\mathbf{g}\right)$$

COMPACTION
$$P^i_\Delta = \phi^i\Delta P^{i*} = -\frac{\eta^i_\phi}{r^i_\phi}\left(\phi^i\nabla\cdot\mathbf{v}^* + \nabla\cdot\mathbf{v}^i_\Delta - \frac{\phi^i}{\rho^i}\frac{D^i\rho^i}{Dt} - \frac{\Gamma^i}{\rho^i}\right)$$

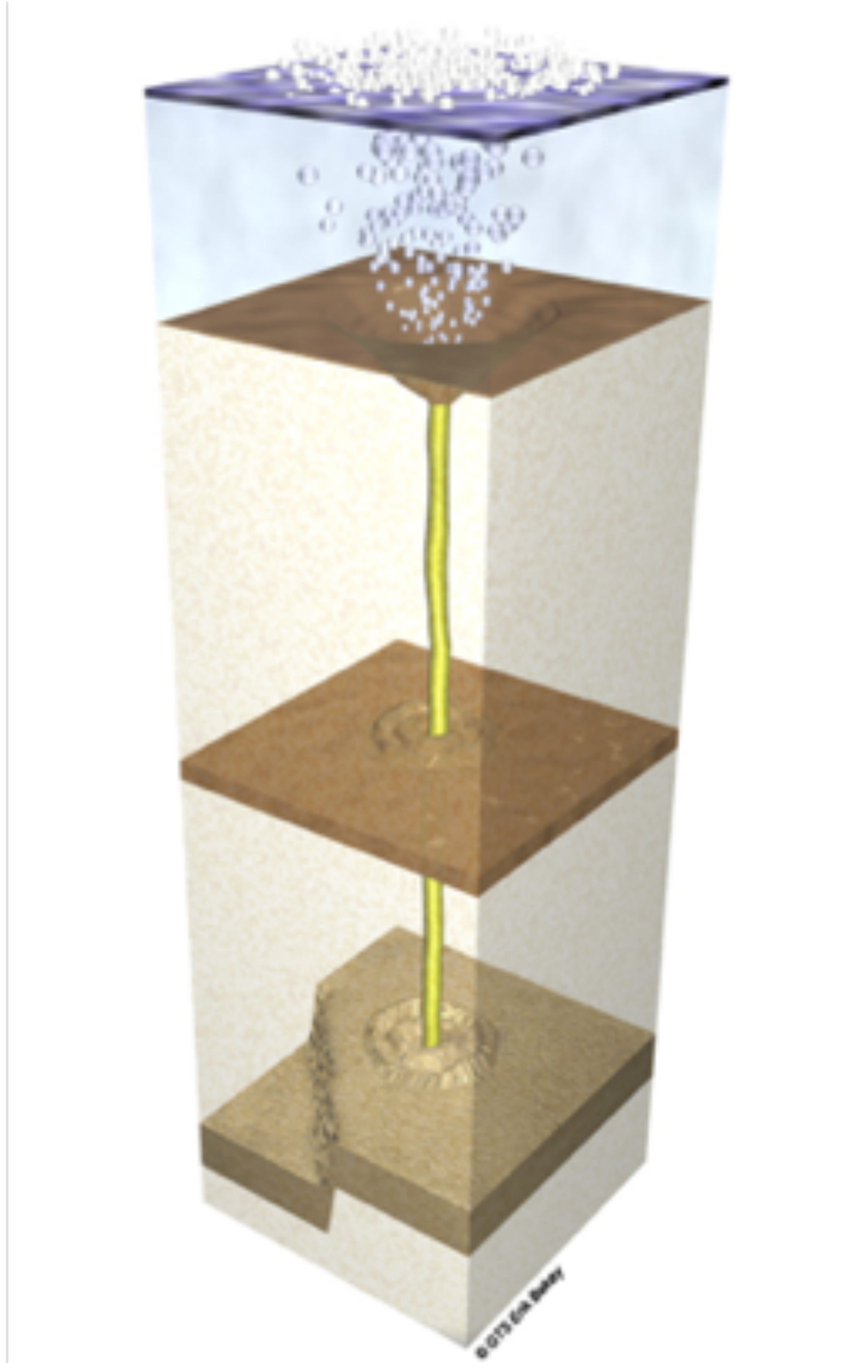
compaction pressure | compaction coefficient | velocity divergences | phase compressibility | mass transfer

Keller and Suckale, in review

Model

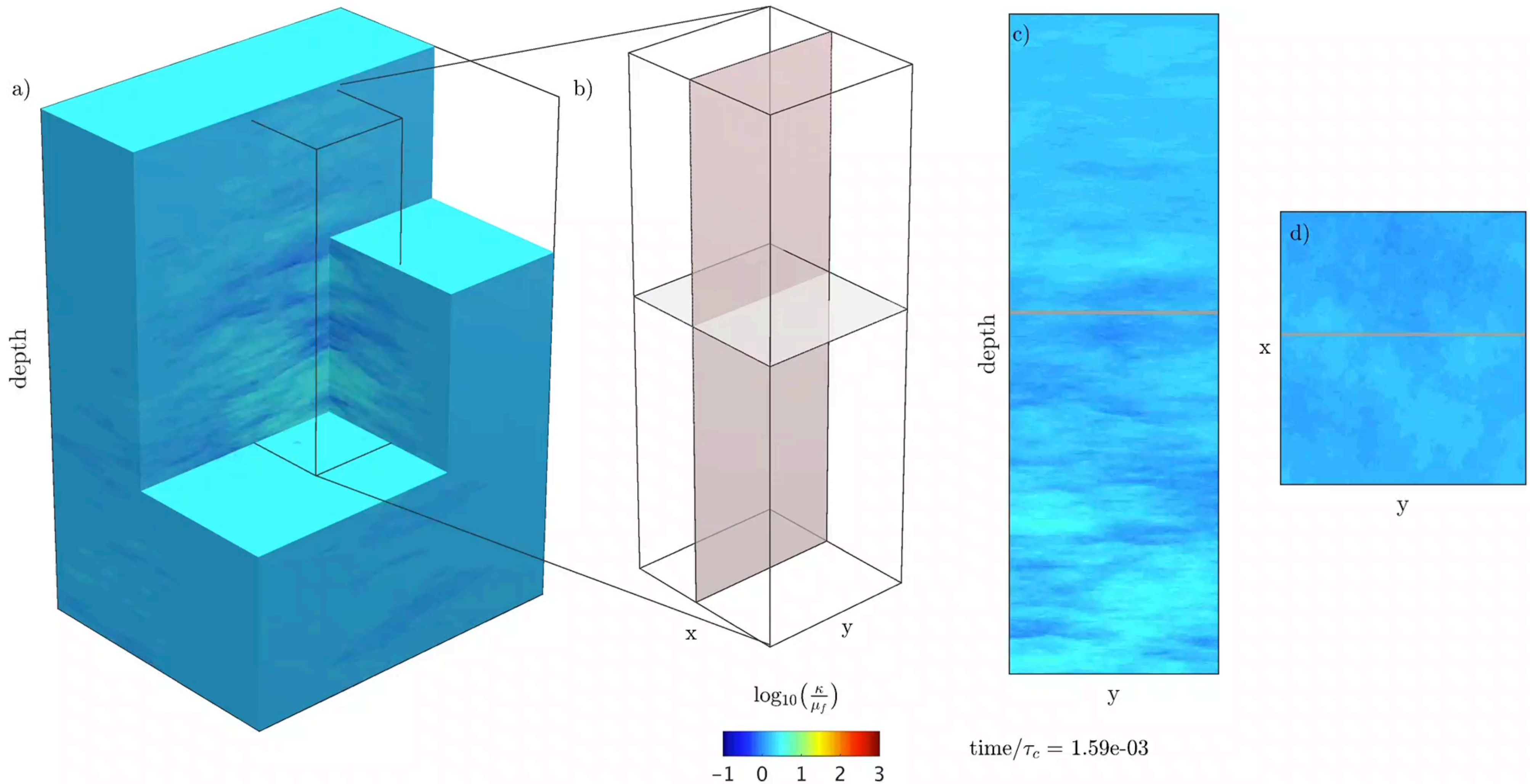


Observation



Ludovic Räss

3D GPU-based code by Ludovic Räss, see Räss et al., 2018, Sketch by Løseth et al, 2011



Simulations by Ludovic Räss, see Räss et al., 2018

Fracture ~~or~~ and? flow?

Sustained flow alters the structural integrity of a rock or granular matrix

FLOW
Stability of continuous rock experiencing sustained flow

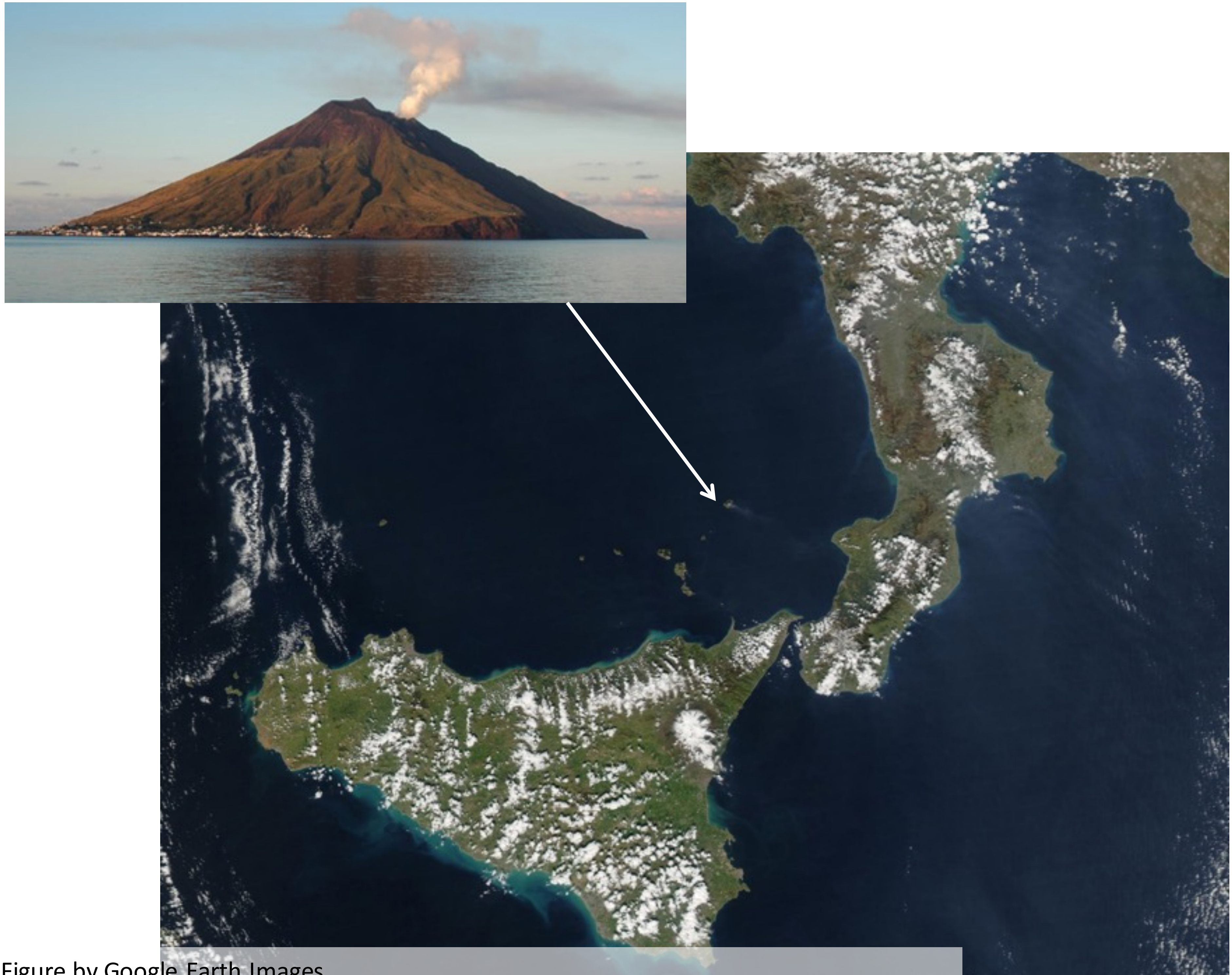
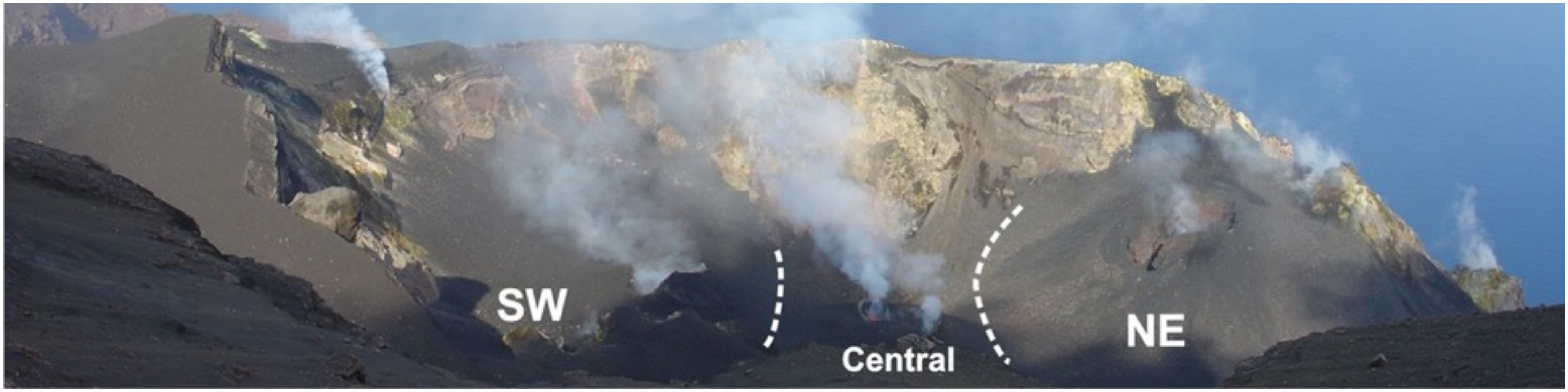
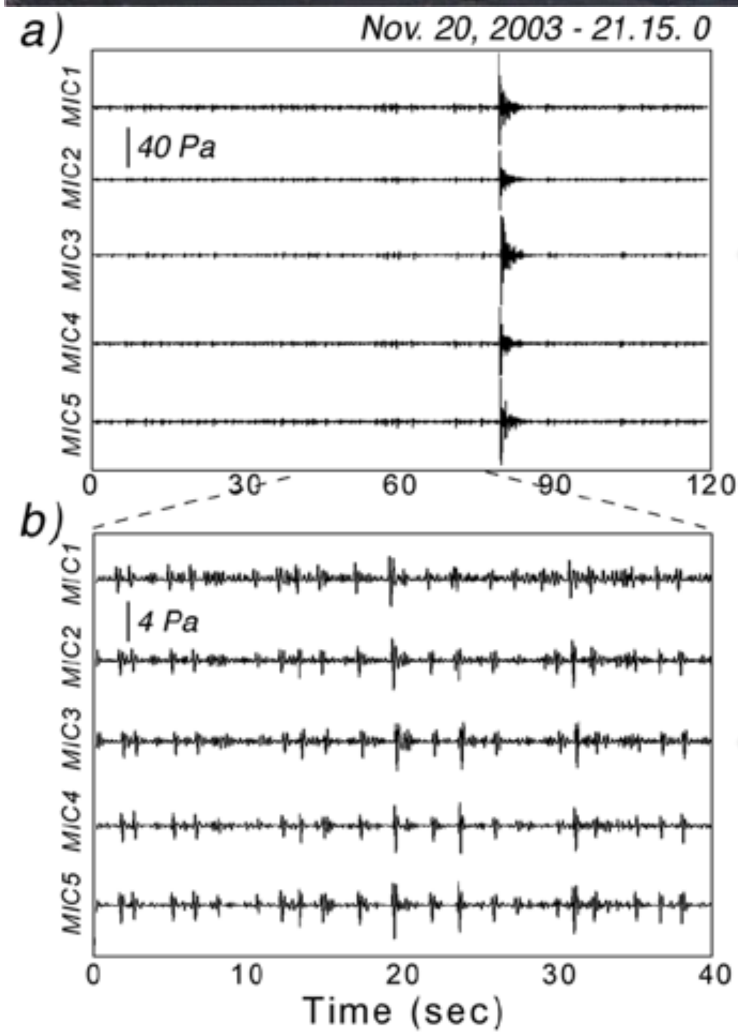


Figure by Google Earth Images


17:35:11



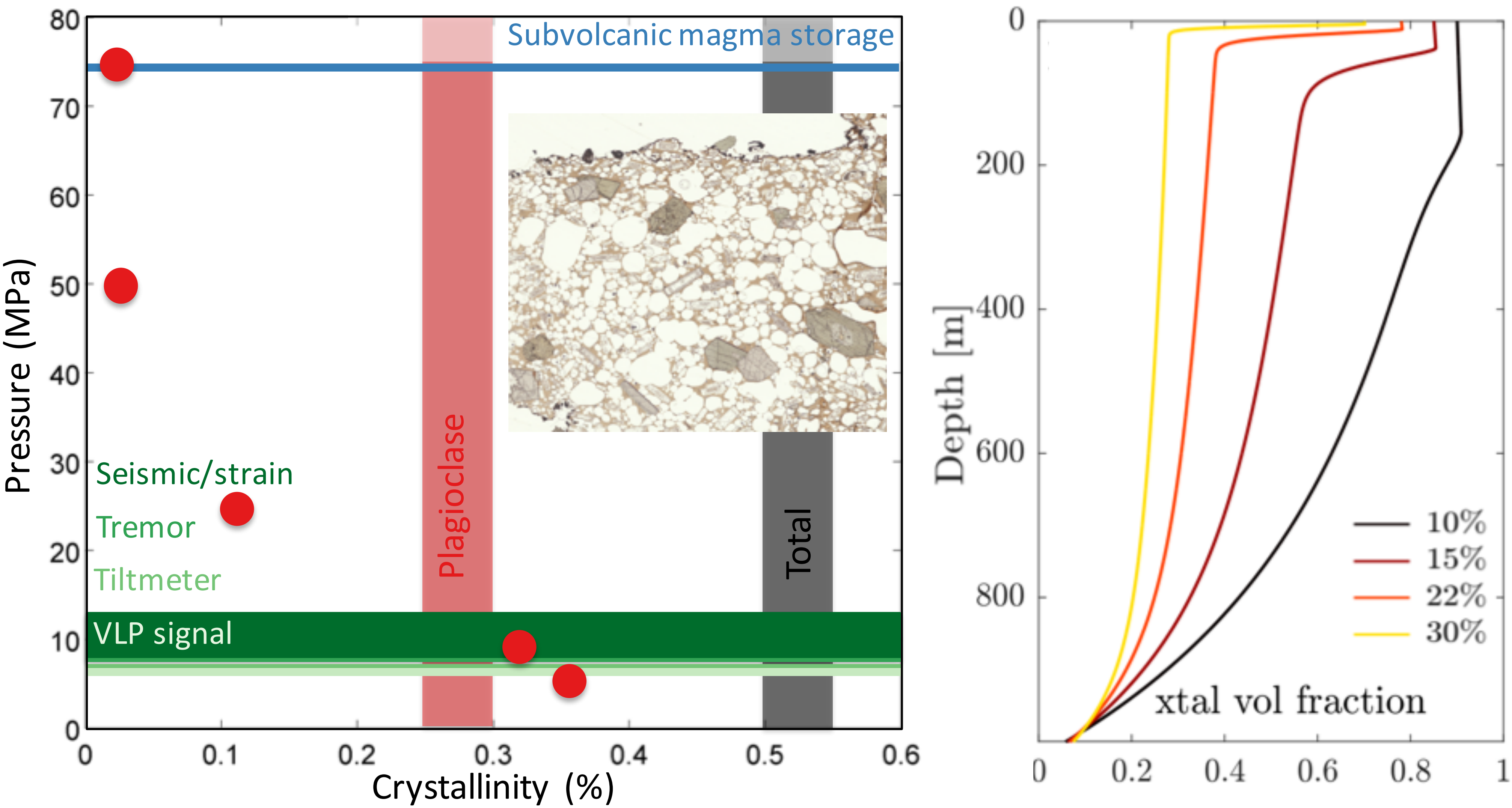


NORMAL STROMBOLIAN EXPLOSIONS: large infrasonic pulses (>10Pa) every 8-10 min

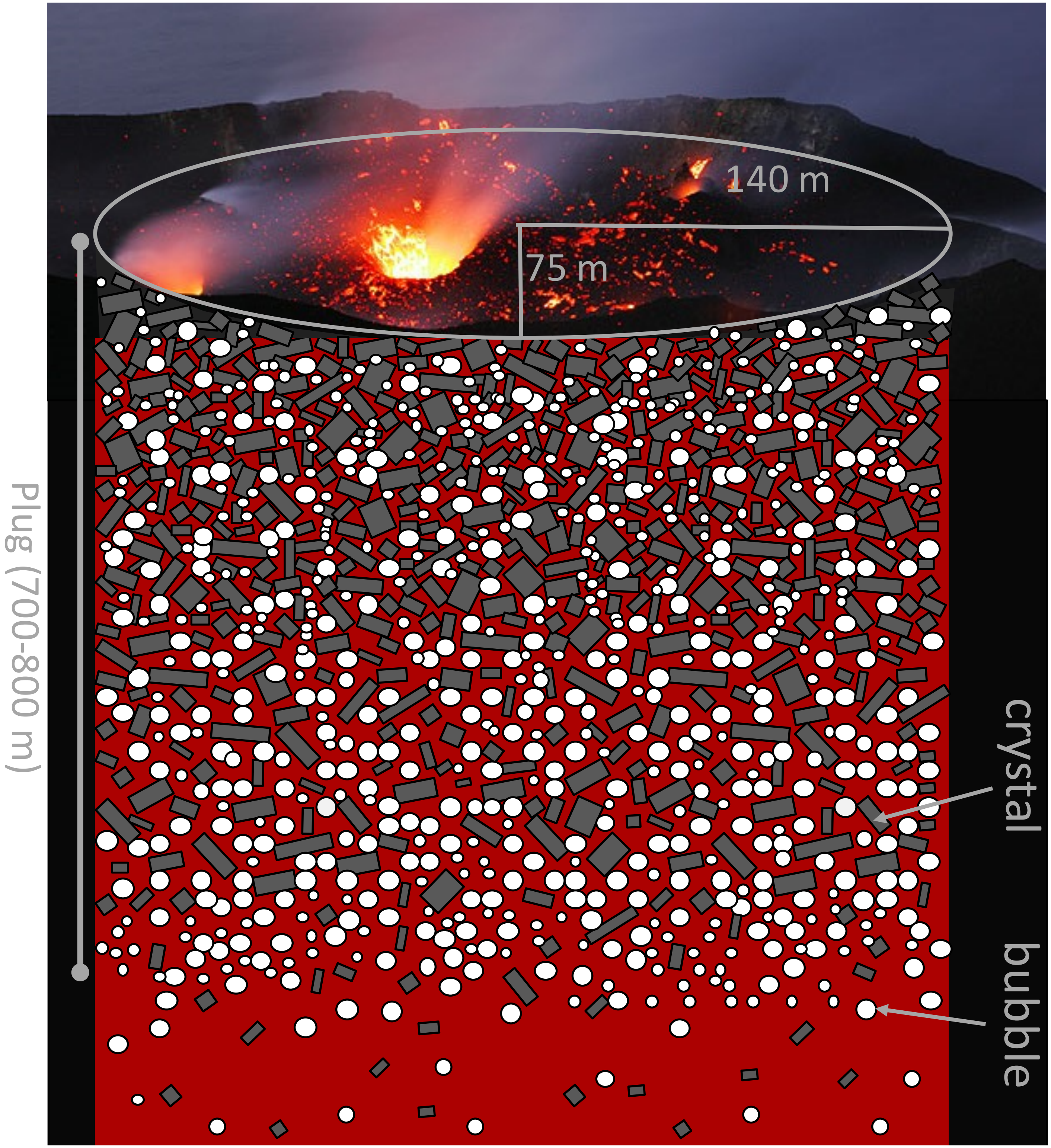
PUFFING: small infrasonic pulses (≈1Pa) every 1-2 s

PASSIVE DEGASSING: continuous and distributed

Top: Figure 14.2 from Rosi et al., 2013; Left: Figure 4 from Ripepe et al., 2007



Experimental crystallinity data from Agostini et al. 2013; Computations from Suckale et al. 2016



140 m
75 m
Plug (700-800 m)
crystal
bubble

Governing equations

Tobias Keller

SEGREGATION
conservation of phase momentum

$$\phi^i \Delta \mathbf{v}^{i*} = -\frac{q_\phi^i}{\eta_\phi^i}\left(\phi^i \nabla P^* + \nabla \phi^i \Delta P^{i*} - \nabla\cdot\phi^i\eta_\phi^i \mathbf{D}\left(\mathbf{v}^i\right) - \phi^i\rho^i\mathbf{g}\right)$$

COMPACTION
conservation of phase mass

$$\phi^i\Delta P^{i*} = -\frac{\eta_\phi^i}{r_\phi^i}\left(\phi^i\nabla\cdot\mathbf{v}^* + \nabla\cdot\phi^i\Delta\mathbf{v}^{i*} - \frac{\phi^i}{\rho^i}\frac{D^i\rho^i}{Dt} - \frac{\Gamma_\rho^i}{\rho^i}\right)$$

PHASE EVOLUTION
conservation of phase volume

$$\frac{D^i\phi^i}{Dt} = \nabla\cdot\kappa_\phi^i\nabla\Delta\phi^{i*} - \phi^i\nabla\cdot\mathbf{v}^i - \frac{\phi^i}{\rho^i}\frac{D^i\rho^i}{Dt} - \frac{\Gamma_\rho^i}{\rho^i}$$

CHEMICAL EVOLUTION
conservation of component mass

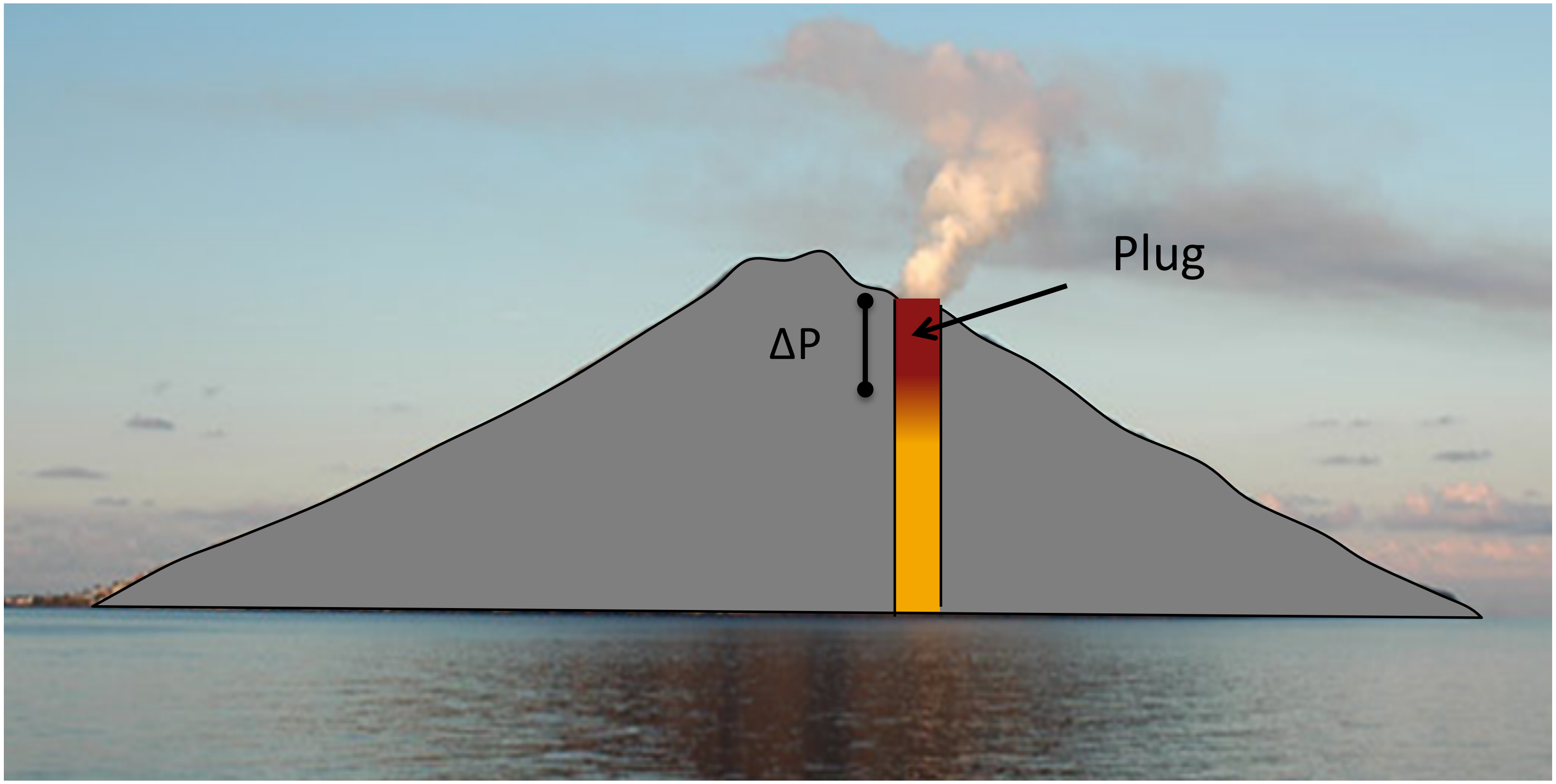
$$\frac{D^i c_j^i}{Dt} = \nabla\cdot\kappa_j^i\nabla\Delta c_{j*}^i + \frac{\Delta c_j^{i\Gamma}\Gamma_\rho^i}{\phi^i\rho^i}$$

THERMAL EVOLUTION
conservation of phase energy

$$\frac{D^iT^i}{Dt} = \nabla\cdot\kappa_T^i\nabla T^i - \frac{\kappa_T^i}{d^{i2}}\Delta T^{i*} + \frac{L^{i*}\Gamma_\rho^i}{\phi^i\rho^i c_P^i} + \frac{\Psi^i}{\phi^i\rho^i c_P^i} + \frac{\alpha^i T^i}{\rho^i c_P^i}\frac{D^iP^i}{Dt} + \frac{H^i}{c_P^i}$$

Keller and Suckale, in review

Regime 1: Darcy flow



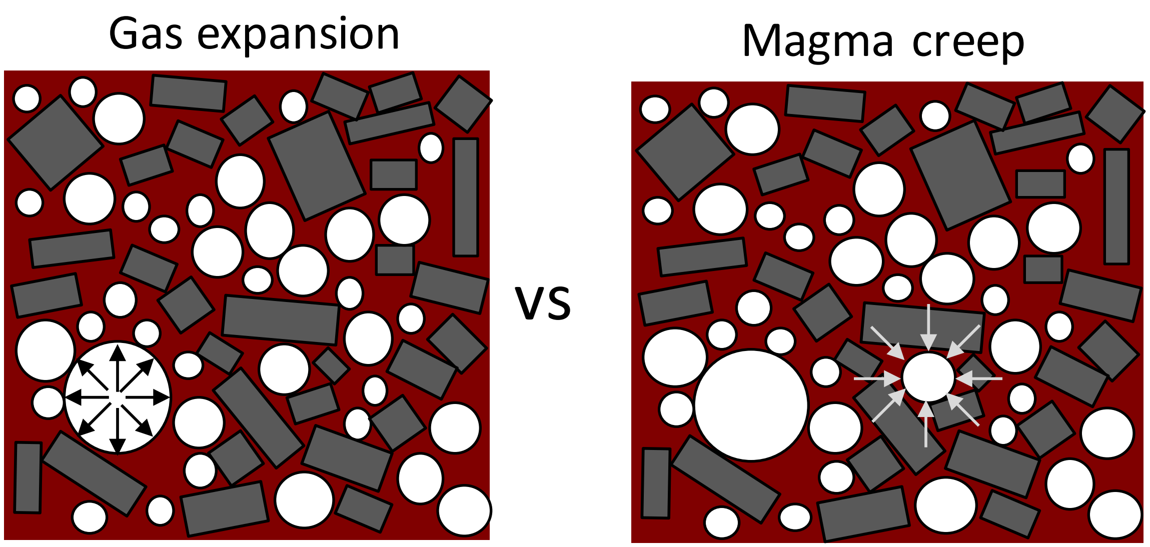
$u_m \ll u_g$ → No overpressure

$$q_g = -\frac{k(T)}{\mu_g} \nabla P$$

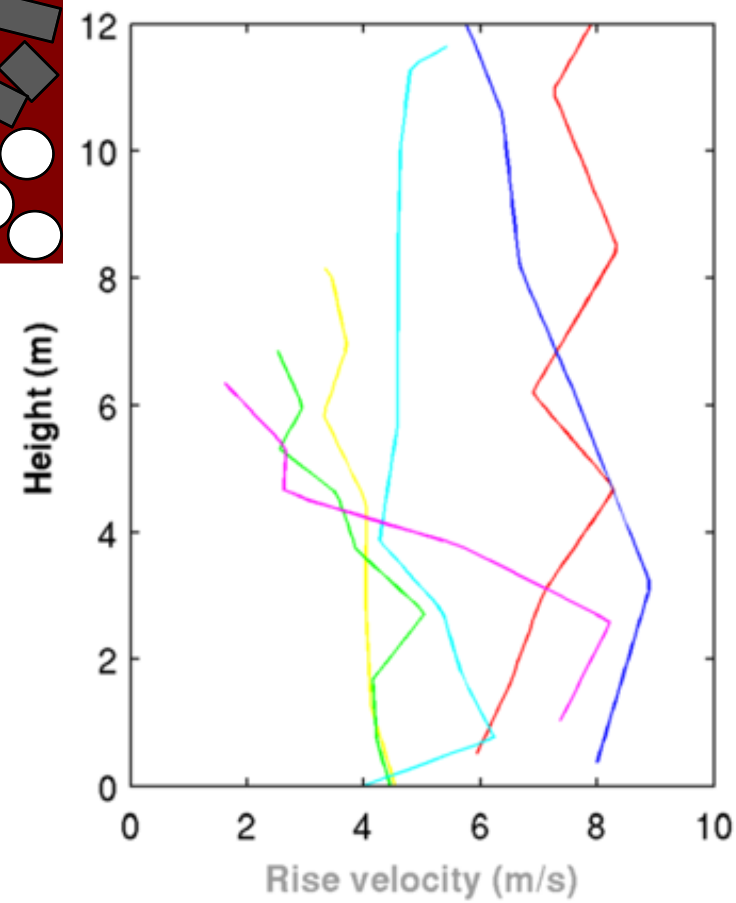
Keller and Suckale, in preparation; Permeability scaling from Bai et al., 2010

Regime 2: Degassing waves

$\lambda \approx 1 - 10\,\mathrm{m}$

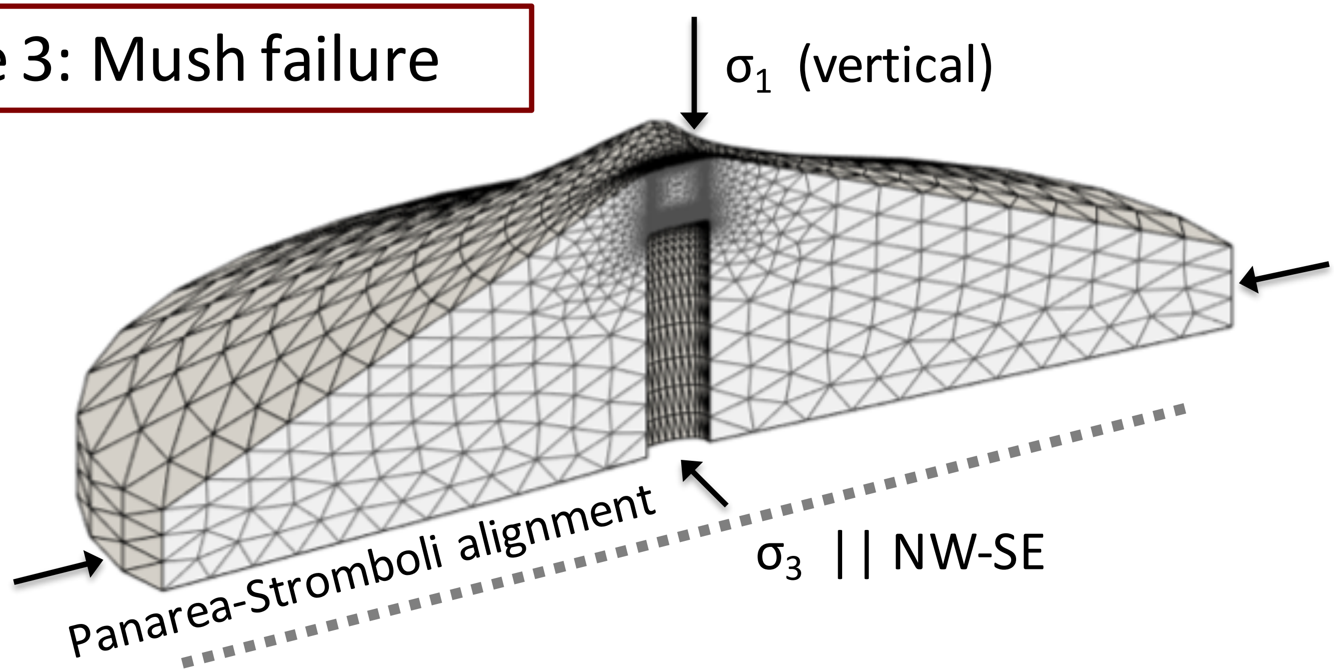


Pressure pulses every 1-2s



See also: Michault et al., 2013; Rise velocity from Gaudin et al., 2017

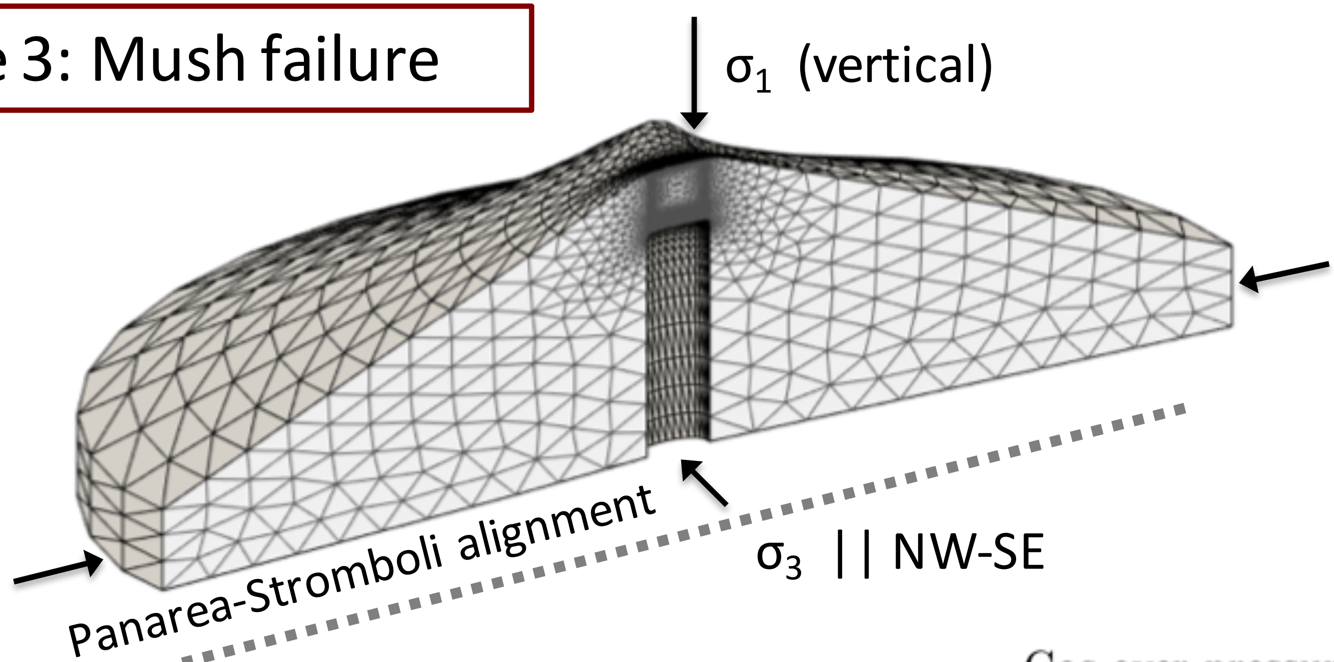
Regime 3: Mush failure



$$\tau_* = f(\sigma_n - p)$$

Suckale et al., 2016

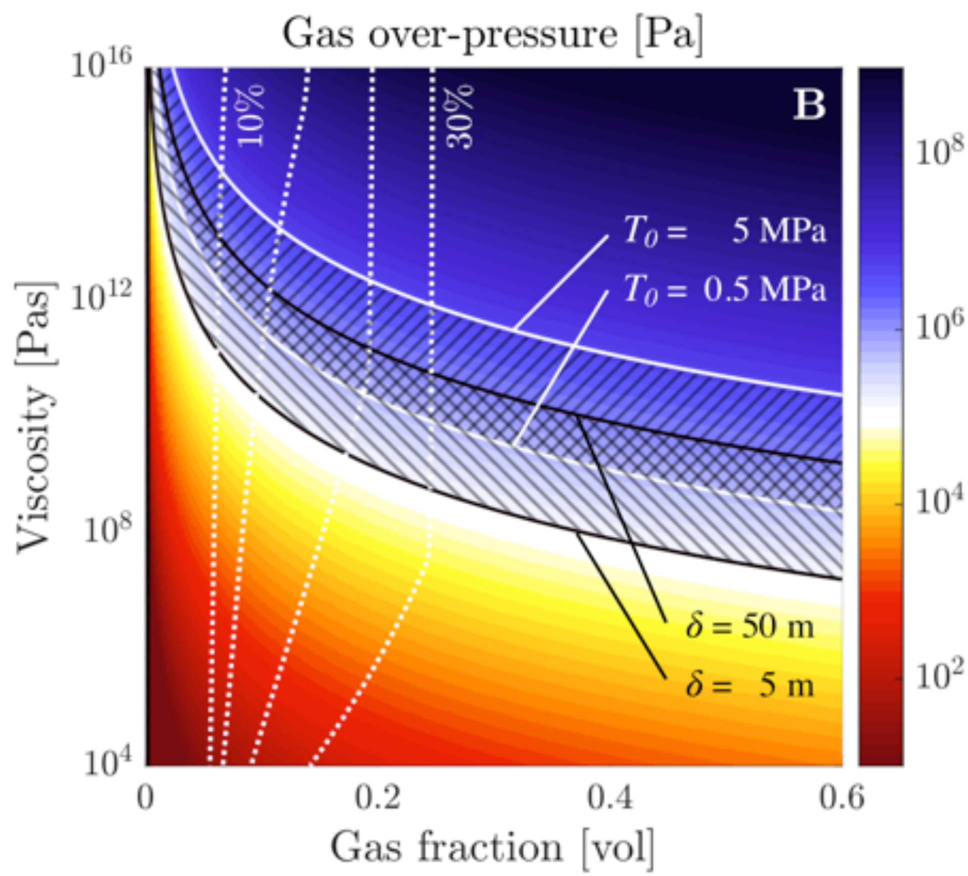
Regime 3: Mush failure



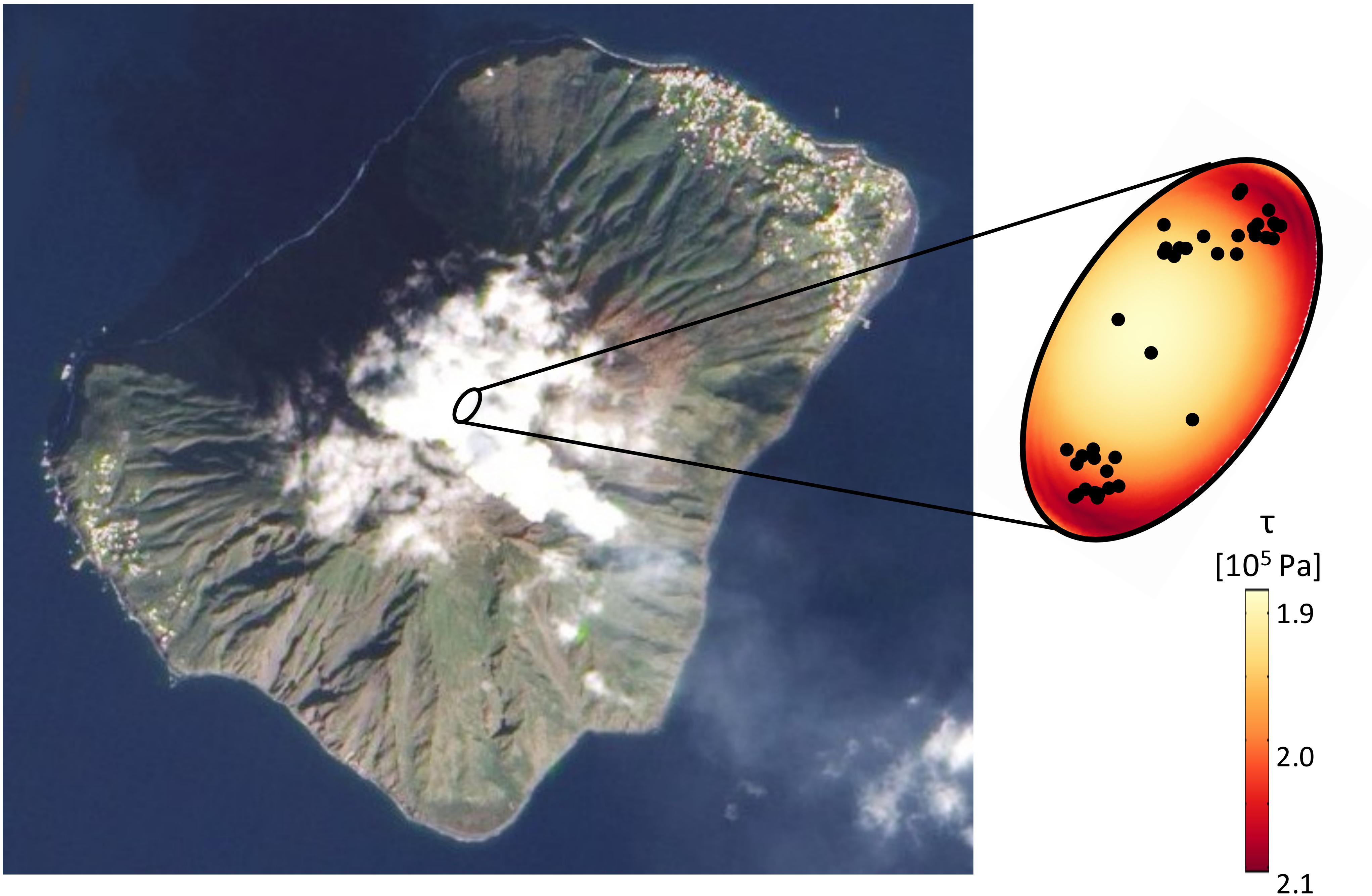
$$\tau_* = f(\sigma_n - p)$$

$$p_c := p_g - p_m = \zeta \nabla \cdot \mathbf{v_m}$$

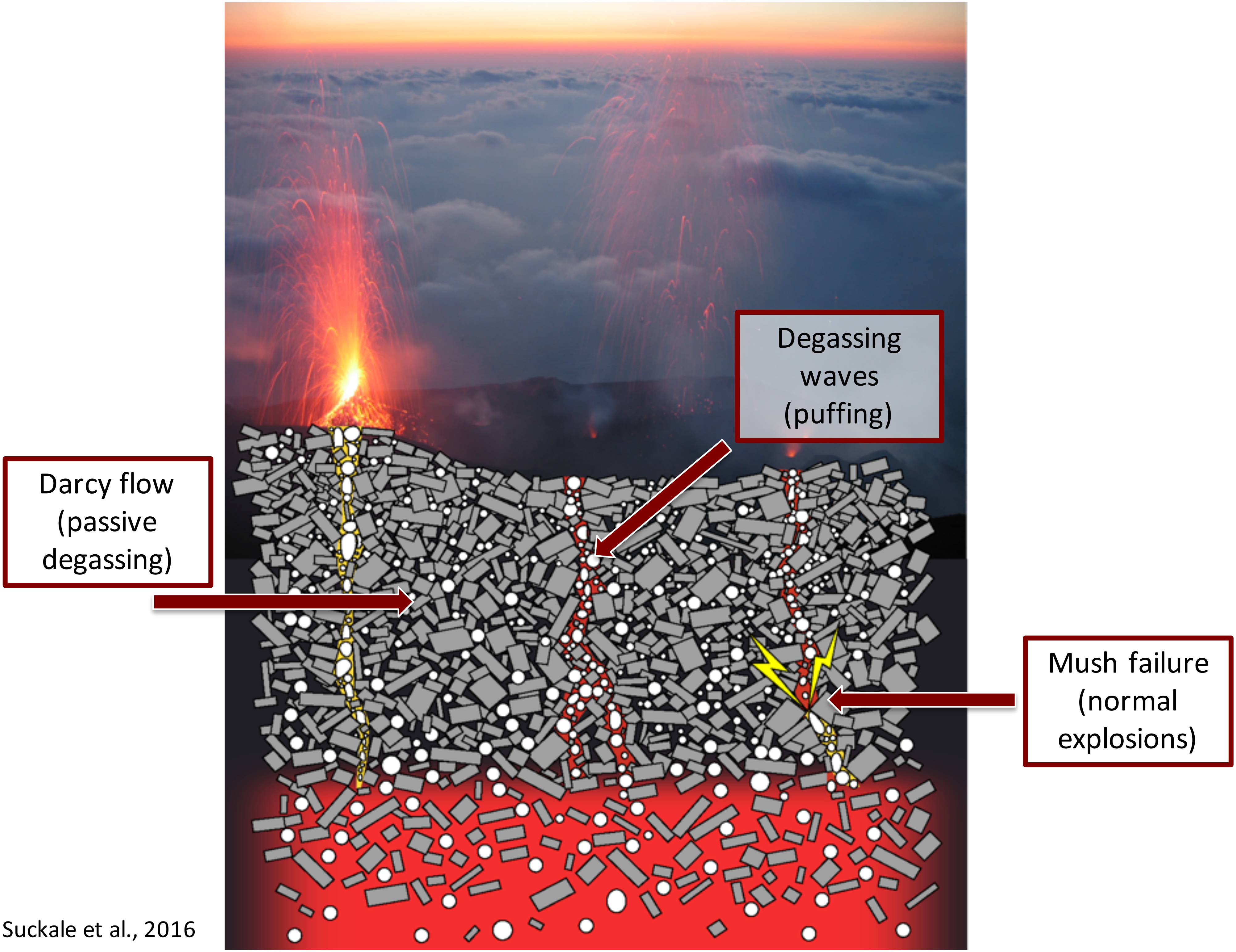
$$\sim \Delta\rho g \delta$$



Suckale et al., 2016



NASA ISS Image: Stromboli Volcano, Italy; Crater locations from Harris et al. 1996; Suckale et al. 2016



Suckale et al., 2016

Fracture ~~or~~ and? flow?

- Sustained flow alters the structural integrity of a rock or granular matrix

- The interplay between flow and matrix deformation results in overpressure and may entail failure

FLOW
Stability of continuous rock experiencing sustained flow


ICE


FIRE


ROCK


WATER

Conclusions:

1) Multiphase interactions at the granular scale can trigger a shift in the system-scale dynamics.

2) Fracture and flow often occur simultaneously and interact more dynamically than we usually give them credit for.

3) Studying extreme event's across different natural systems with a similar model increases opportunities to refute/validate models.

Thank you.

Look deep into nature and you will understand everything better.
– Albert Einstein

SIGMA @ GitLab

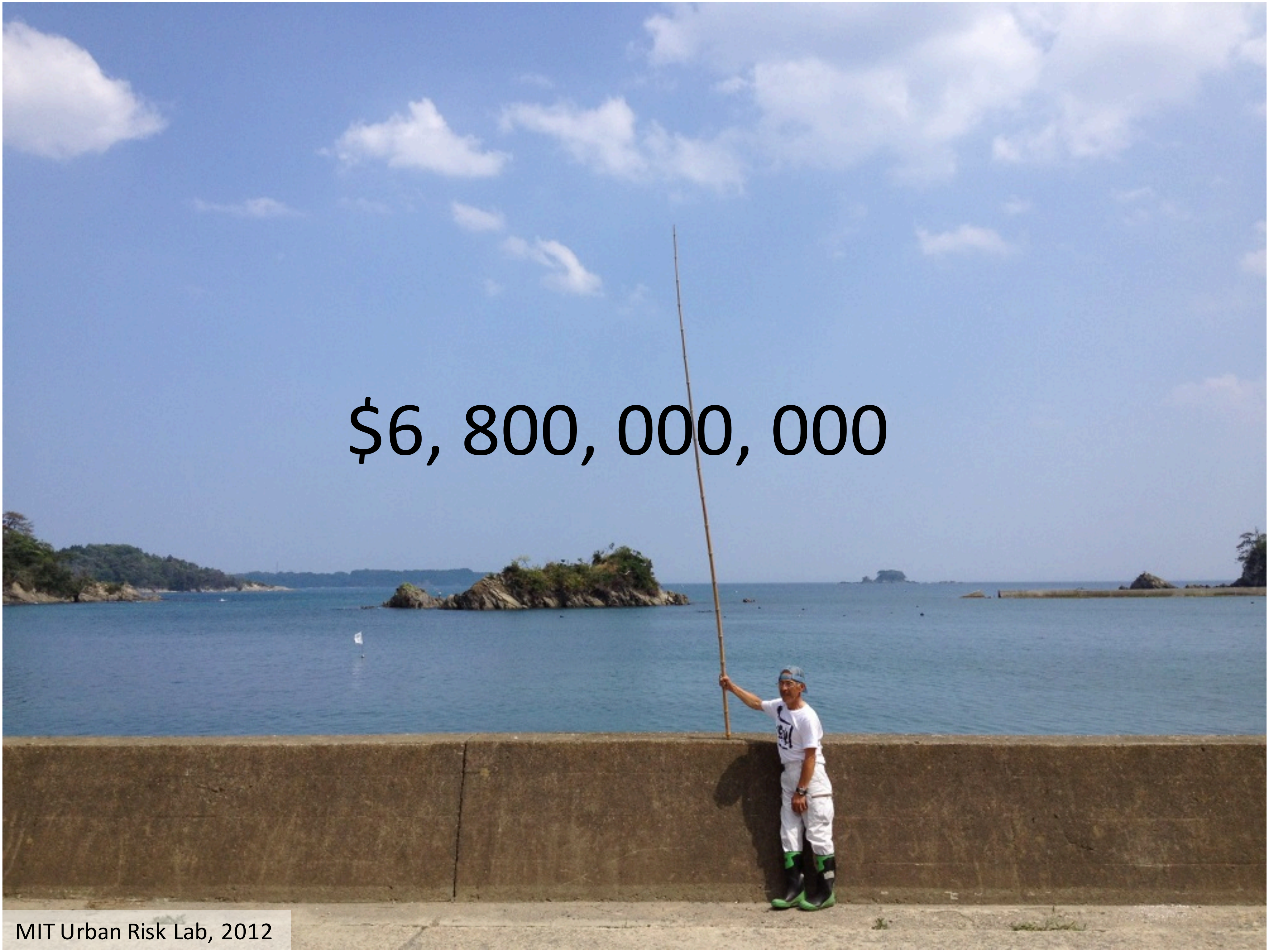

Minamisanriku, Japan:
95 % buildings destroyed
60% population missing
40% evacuation sites flooded

100 out of 300 km of sea walls in Tohoku destroyed.

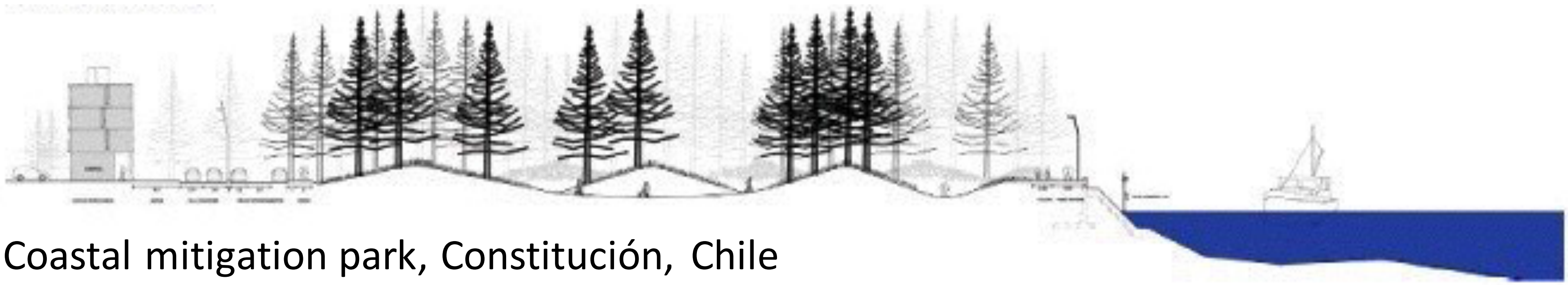
Bottom: Arahama, Miyagi Prefecture; Top: Yamada, Iwate Prefacture

Tohoku in dark green, source: Wiki

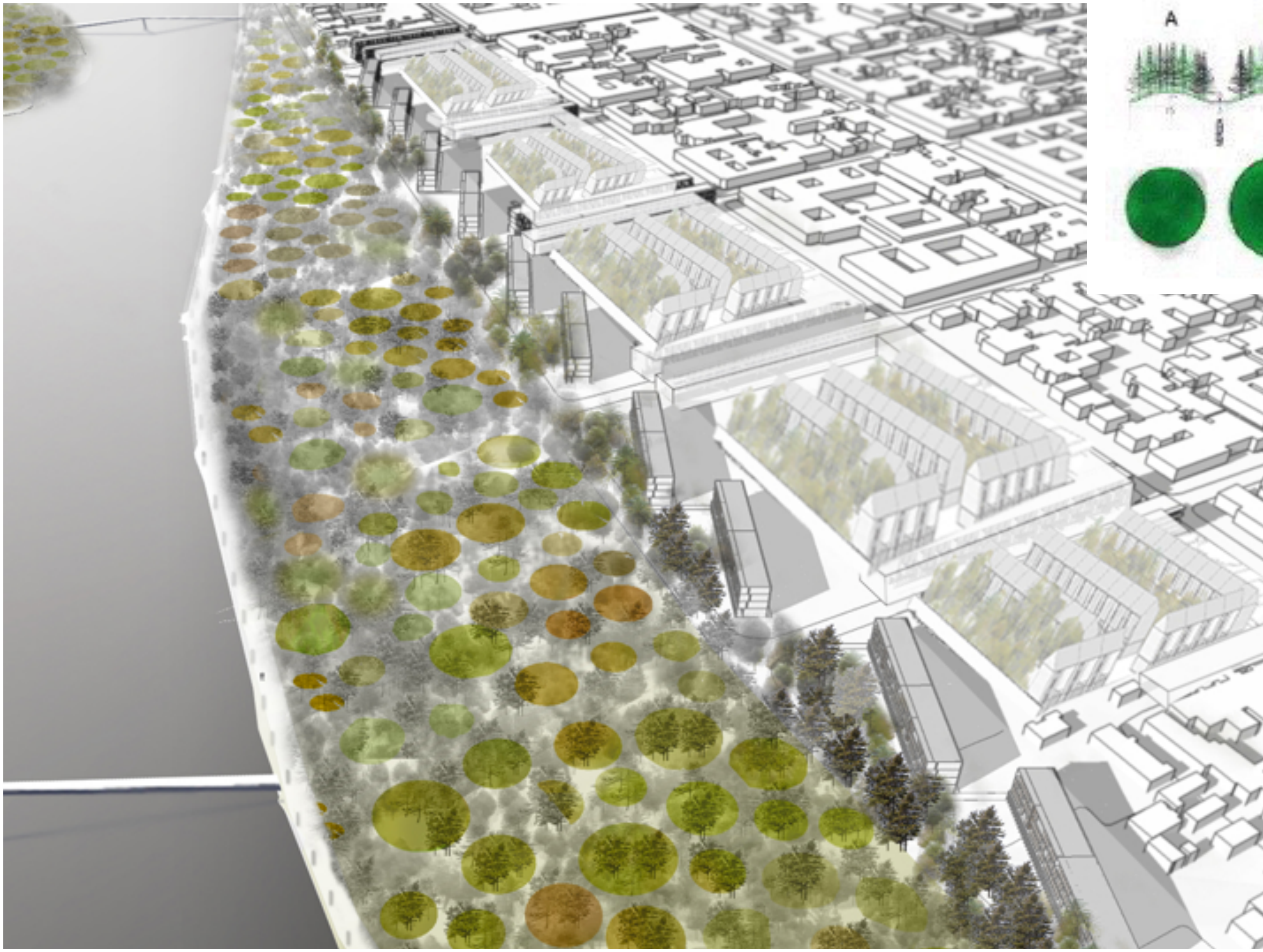

MIT Urban Risk Lab, 2012

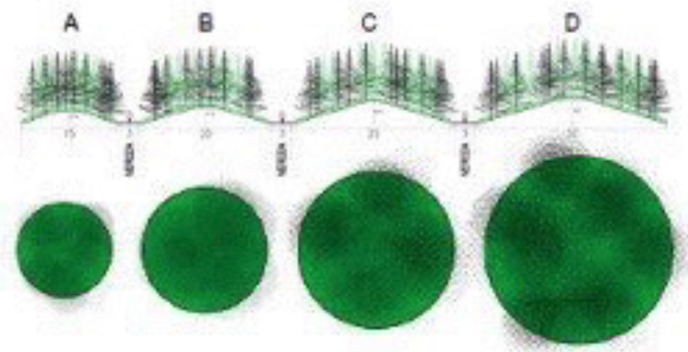

$6, 800, 000, 000
MIT Urban Risk Lab, 2012

70 000 pine trees in Rikuzentakata destroyed, except for one “miracle pine”.

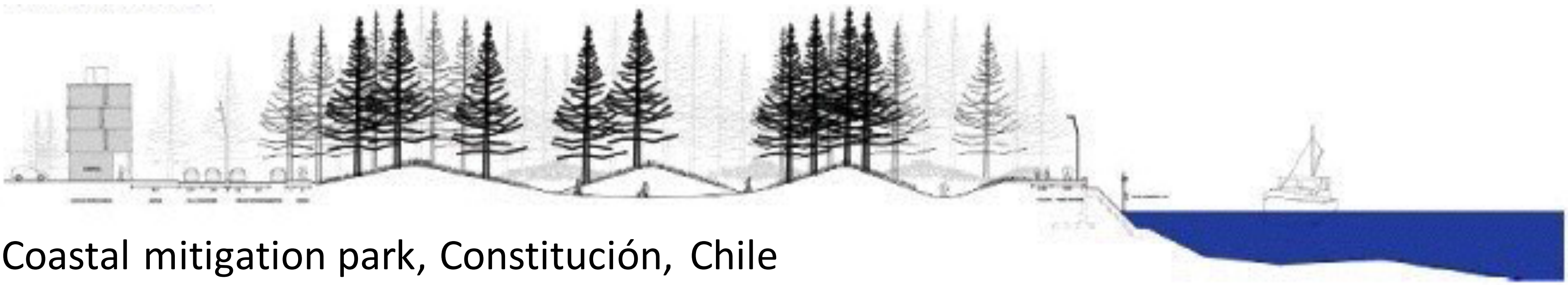


Coastal mitigation park, Constitución, Chile





Hill sizes:
A: 2.0 x 15m
B: 2.6 x 20m
C: 3.4 x 25m
D: 4.0 x 30m



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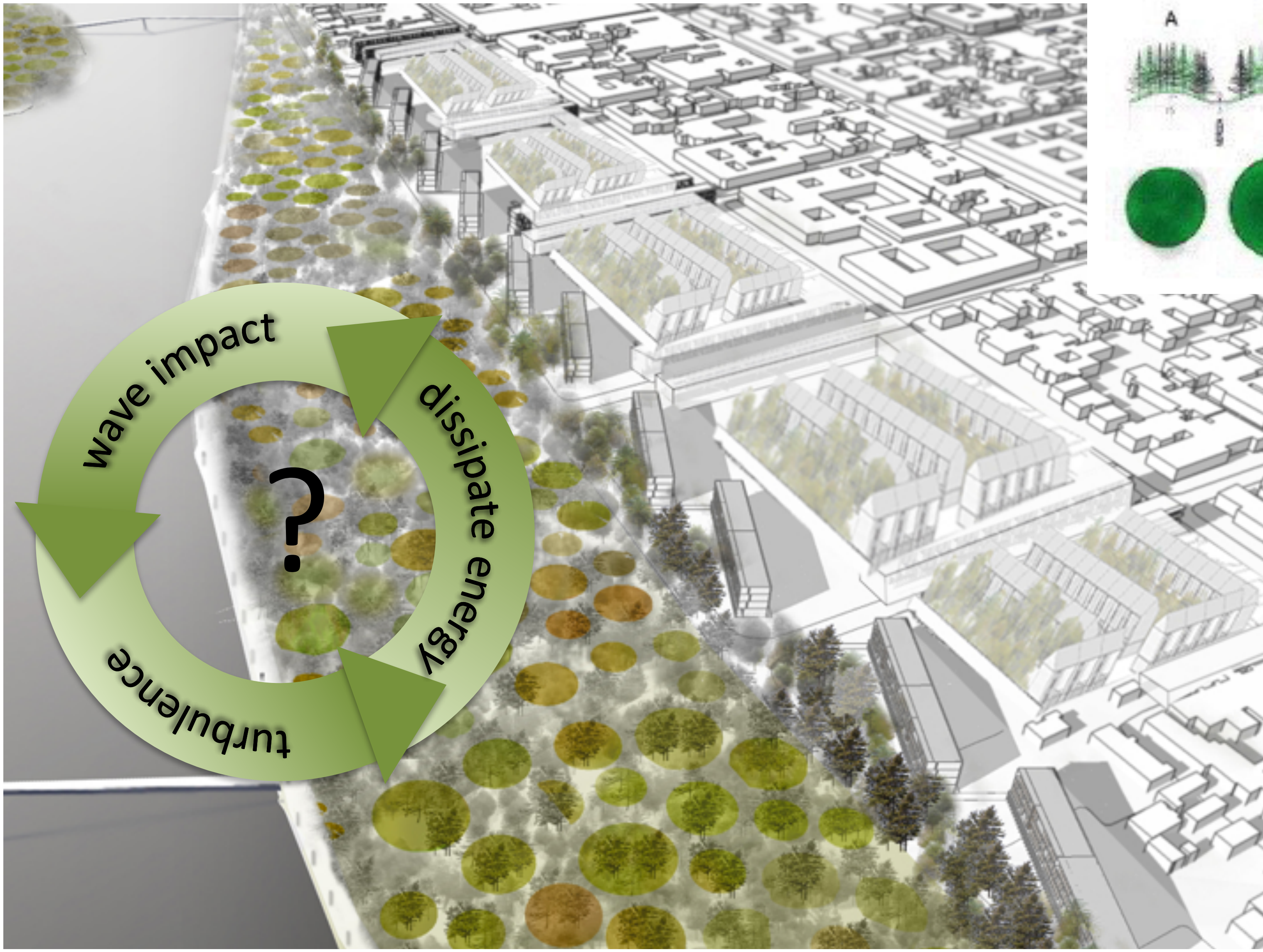

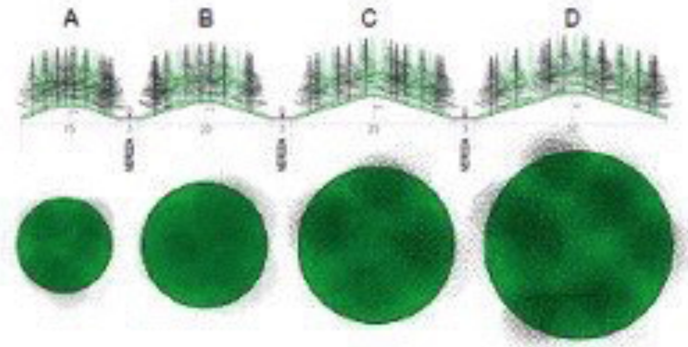

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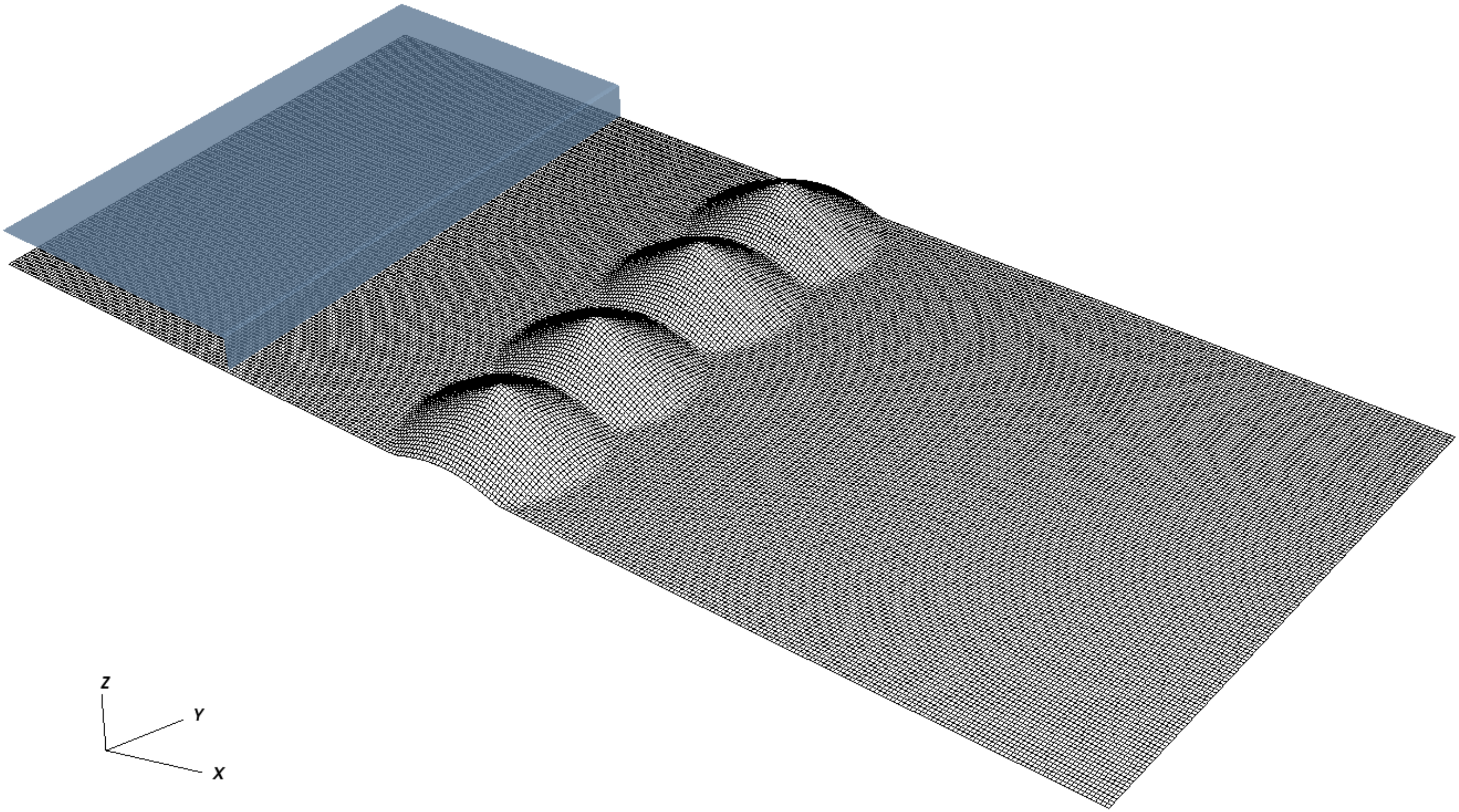


Marras et al., in preparation

Viscous shallow water equations (NUMA2D)

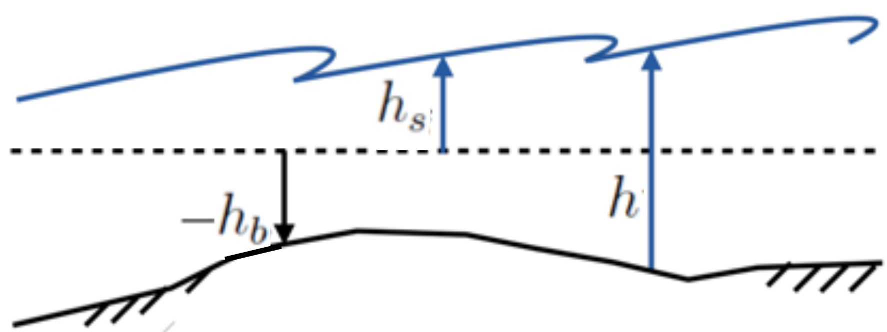
Continuity:
$$\frac{\partial h}{\partial t} + \nabla \cdot (h\mathbf{u}) = \delta \nabla \cdot (\mu_{SGS} \nabla h)$$

Switches viscosity on/off

Momentum:
$$\frac{\partial h\mathbf{u}}{\partial t} + \nabla \cdot \left(h\mathbf{u} \otimes \mathbf{u} + \frac{g}{2}(h^2 - h_b^2) I\right) + g h_s \nabla \cdot (h_b I) = \nabla \cdot (h_s \mu_{SGS} \nabla \mathbf{u})$$

Bathymetry | Free surface | Dynamic dissipation

Dynamic dissipation coefficient:
$$\mu_{SGS} = \max(0, \;\; \min(\mu_{\max}, \mu_{\mathrm{res}}))$$

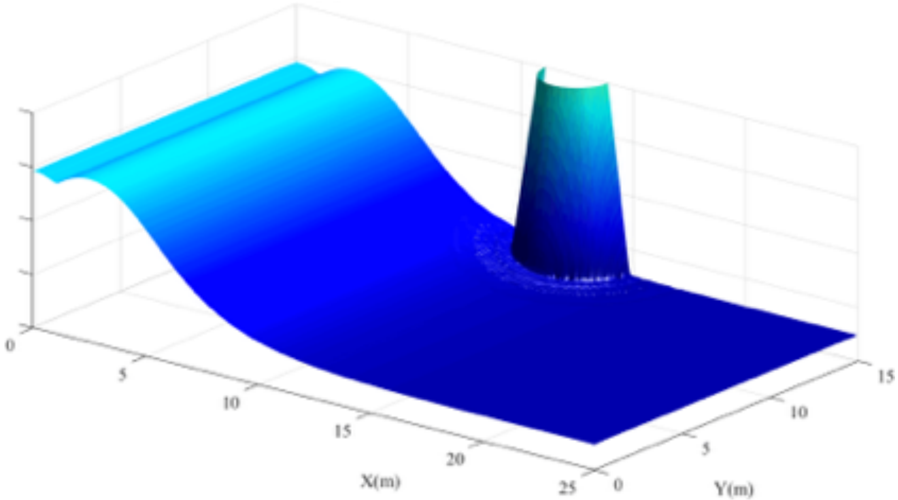


Wetting&drying: 0.1 m threshold

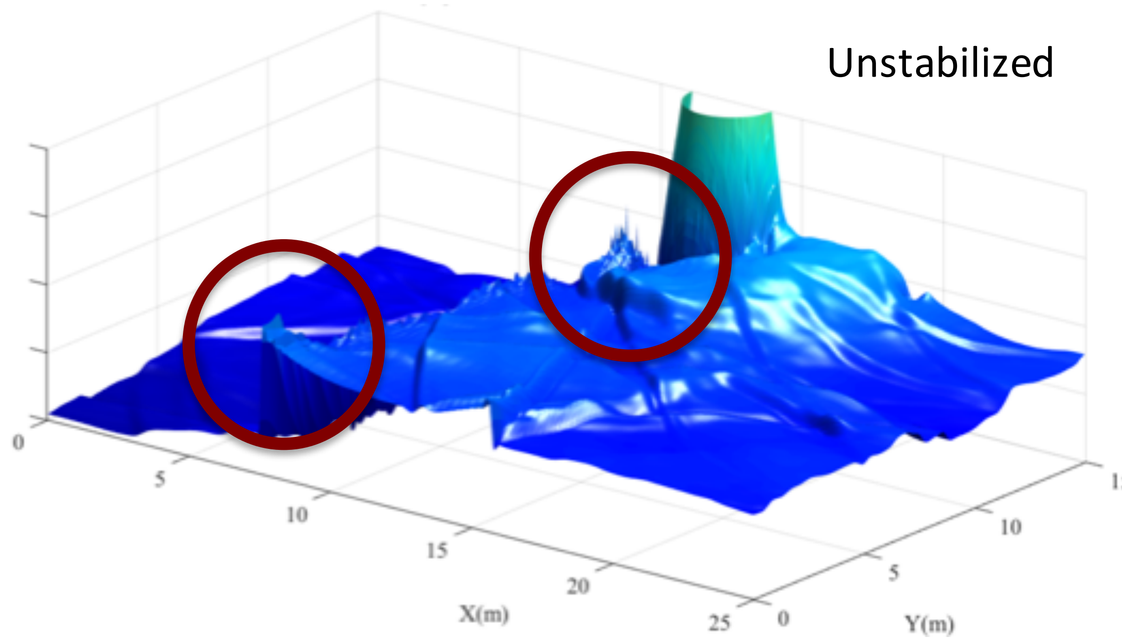
Giraldo et al., 2002; Abdi and Giraldo, 2016; Marras et al., in review

2D solitary wave runup on circular island

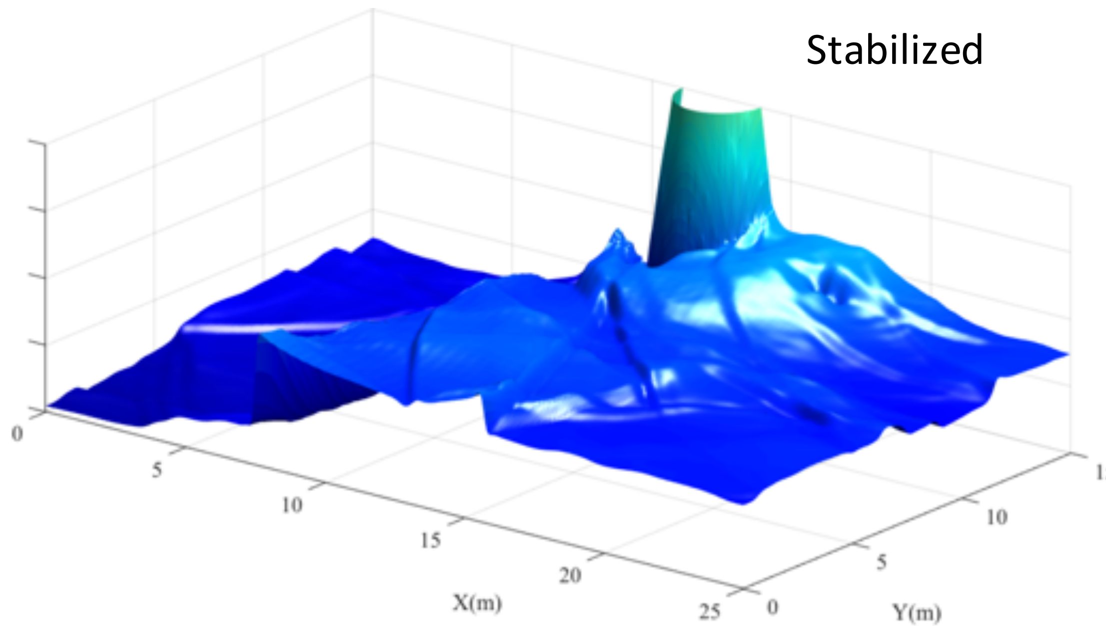
Initial condition



Unstabilized

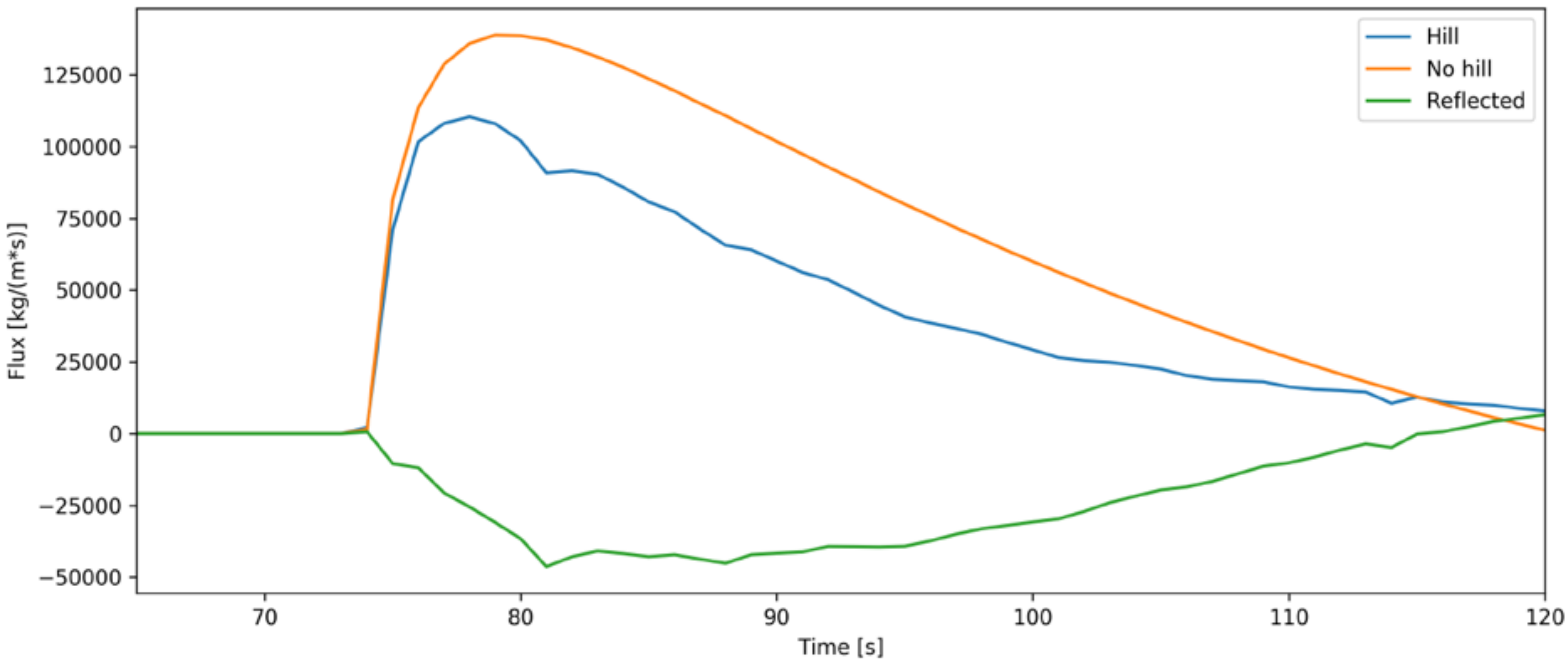


Stabilized



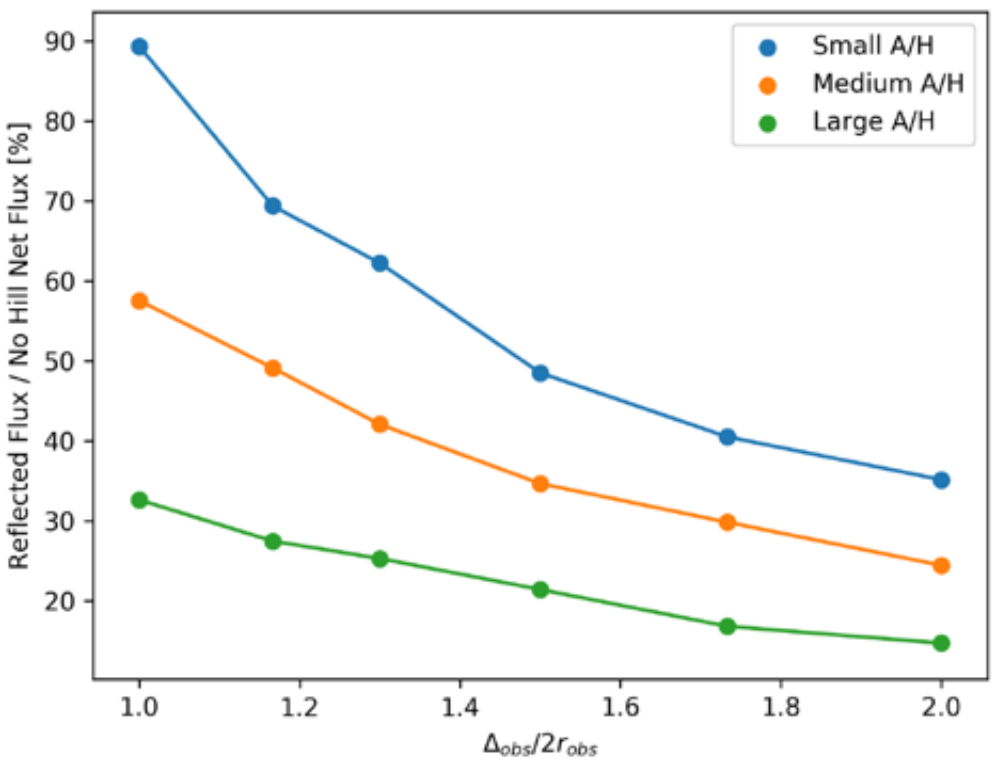
Synolakis, 1987; Marras et al., in review

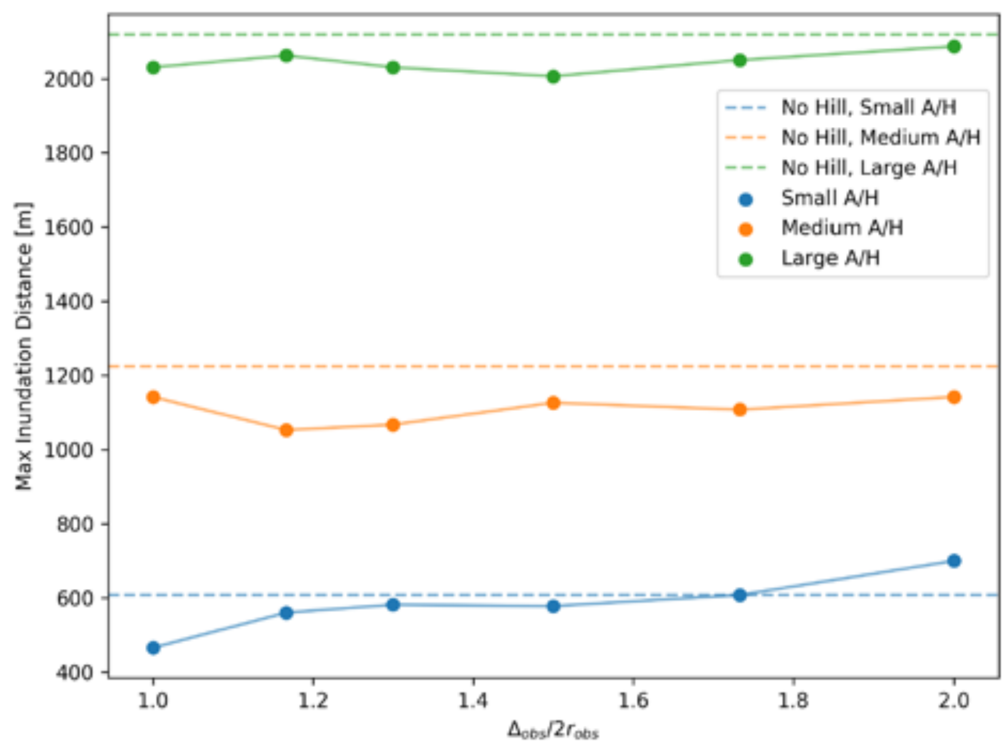
$$F = \int_0^{y_{\max}} \rho u(x = x_{obs}) \cdot h \, dy$$



Lunghino et al., in preparation; Carrier et al., 2003

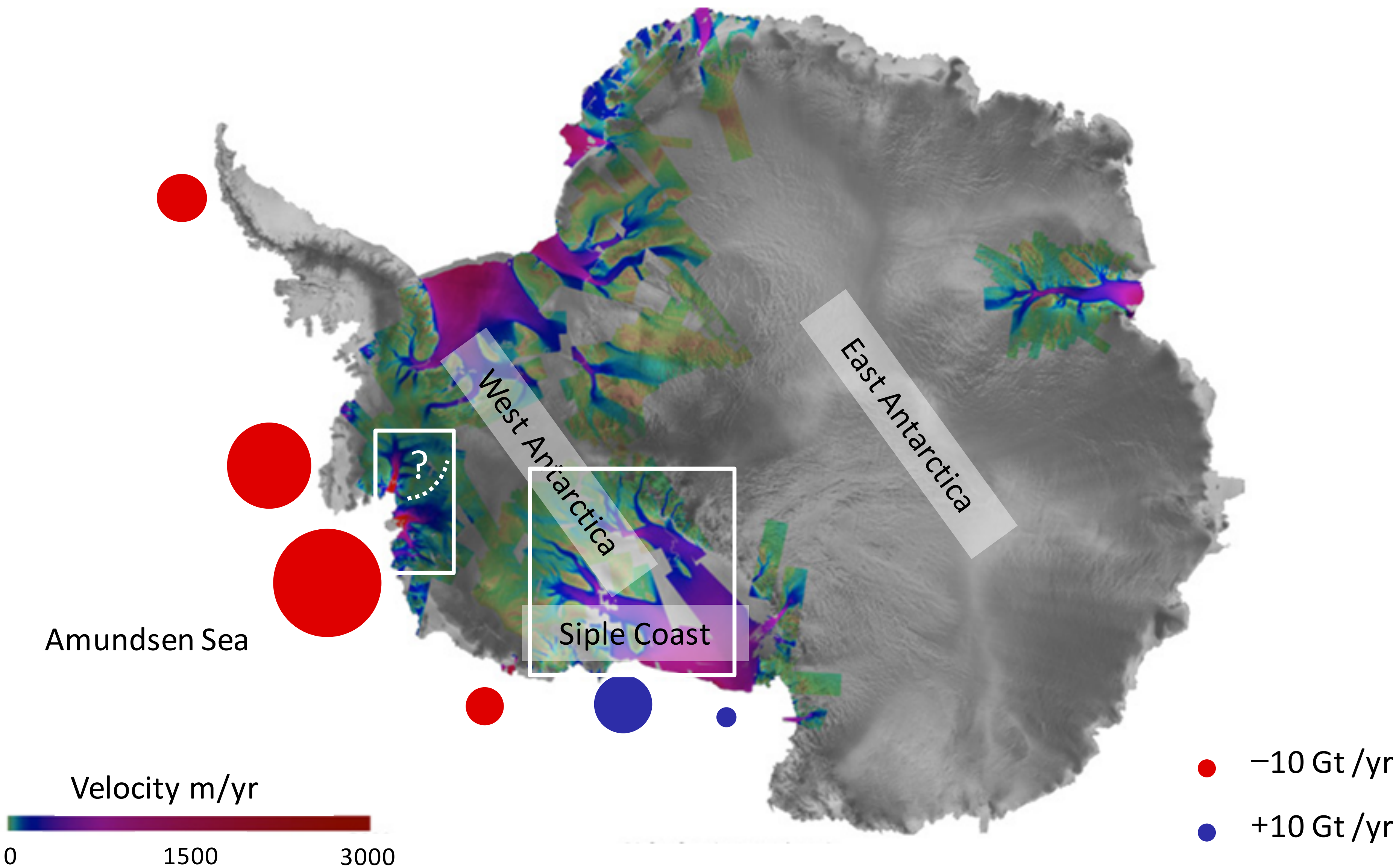
$$F = \int_0^{y_{\max}} \rho u(x = x_{obs}) \cdot h \, dy$$





Lunghino et al., in preparation; Carrier et al., 2003


$\tau_* = f(\sigma_n - p)$
wave impact
dissipate energy
entrainment
?



Eric Rignot, NASA Jet Propulsion Laboratory, Paolo et al., 2015; MacGregor et al., 2013

Fluid solver

$$\rho(\boldsymbol{x})\left(\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right) = -\nabla p + \nabla\cdot\left[\mu(\boldsymbol{x})(\nabla\boldsymbol{v} + \nabla\boldsymbol{v}^{\mathrm{T}})\right] + \boldsymbol{g}\rho(\boldsymbol{x}) - \sigma\kappa(\boldsymbol{x})\delta(\boldsymbol{x})\boldsymbol{n}$$

$$\nabla\cdot\boldsymbol{v} = 0$$

$$[p] = \sigma\kappa + 2[\mu](\nabla u\cdot\boldsymbol{n}, \nabla v\cdot\boldsymbol{n}, \nabla w\cdot\boldsymbol{n})\cdot\boldsymbol{n}$$

Interface solver

$$\frac{\partial\phi}{\partial t} + \mathbf{v}\cdot\nabla\phi = 0$$

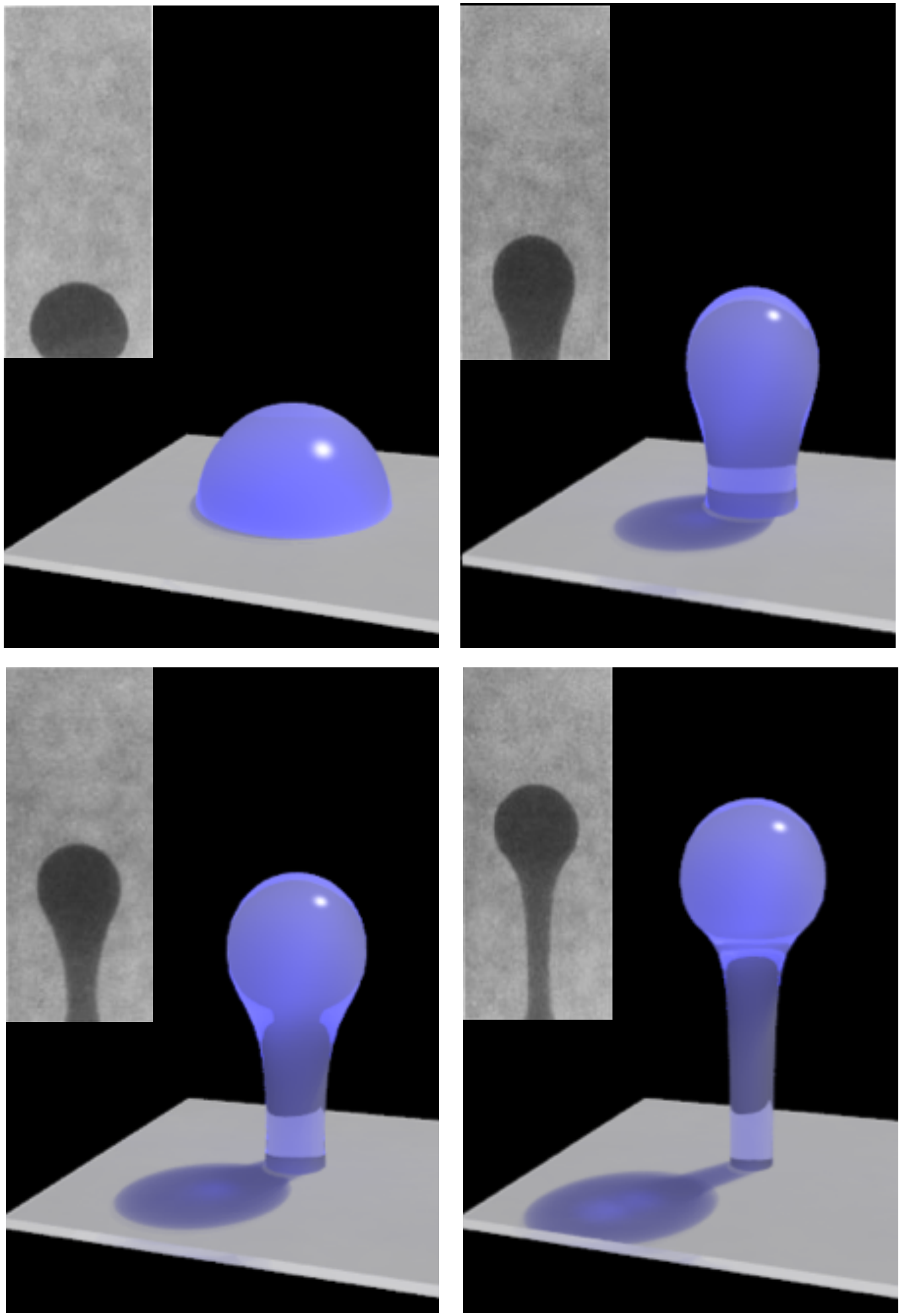
Solid solver

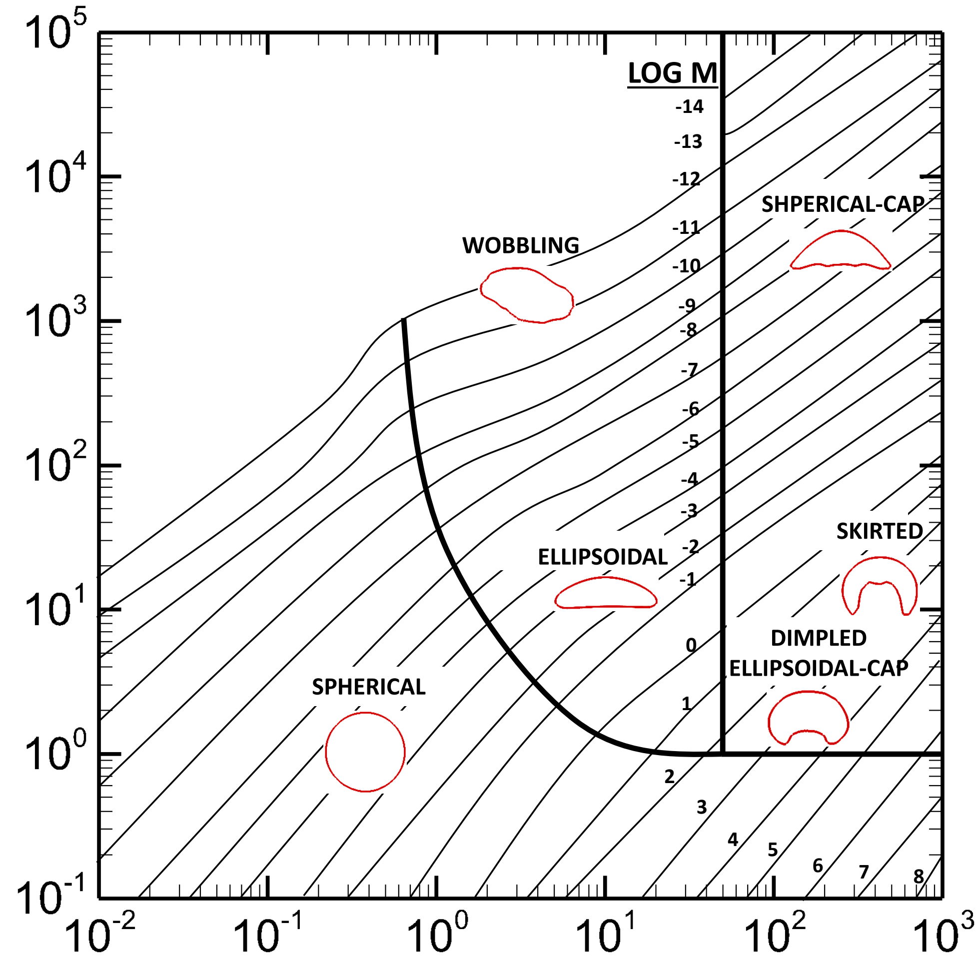
$$\frac{\mathrm{d}(m_p\vec{U}_p)}{\mathrm{d}t} = \vec{F}_p + \vec{g}$$

$$\frac{\mathrm{d}(\hat{I}_p\cdot\vec{\omega}_p)}{\mathrm{d}t} = \vec{T}_p\,,$$

$$\frac{\mathrm{d}\vec{X}_p}{\mathrm{d}t} = \vec{u}_p$$

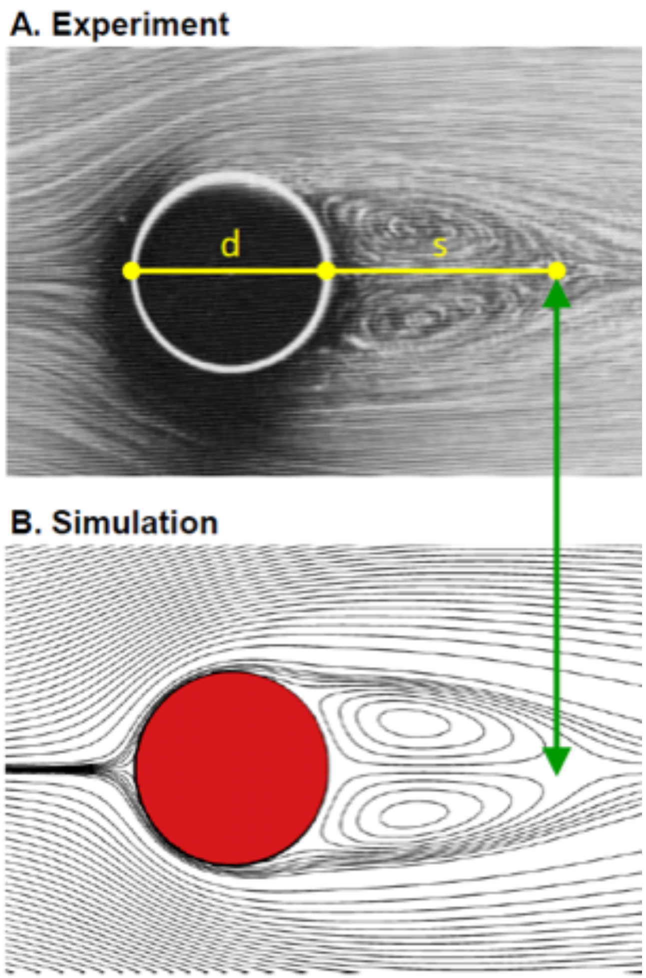
Qin and Suckale, 2017; Suckale et al., 2010a,b; Suckale et al., 2012a



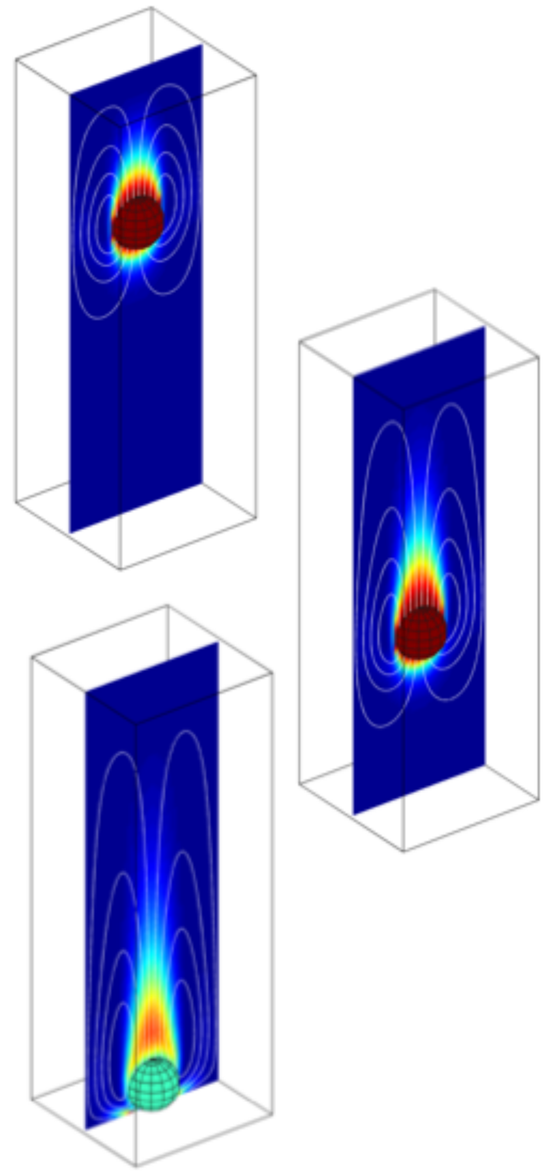


Manga and Stone, 1993; Suckale et al., 2010a; Clift et al., 2005; Qin and Suckale, 2017.

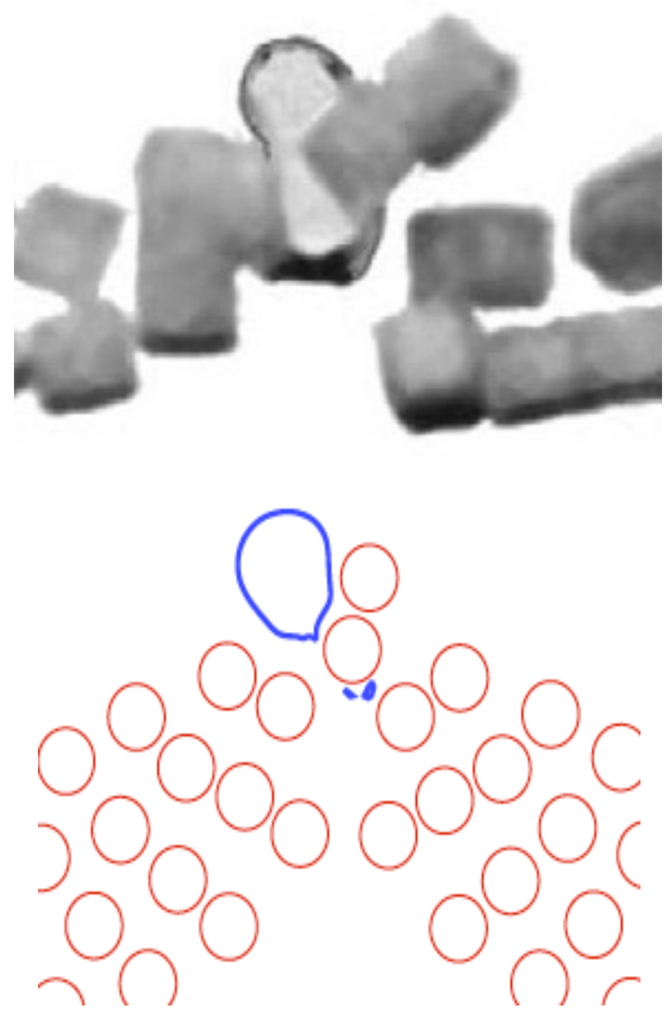
1. Wake formation



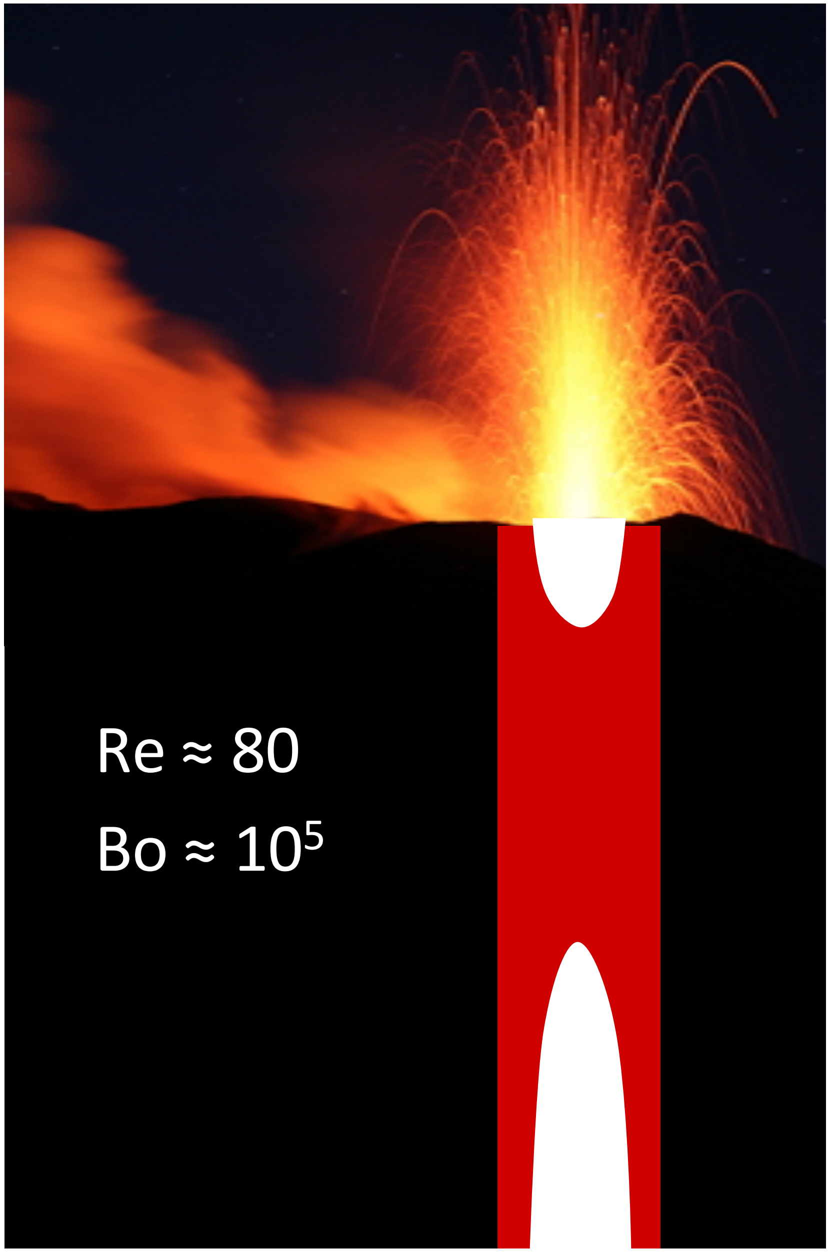
2. Settling speed

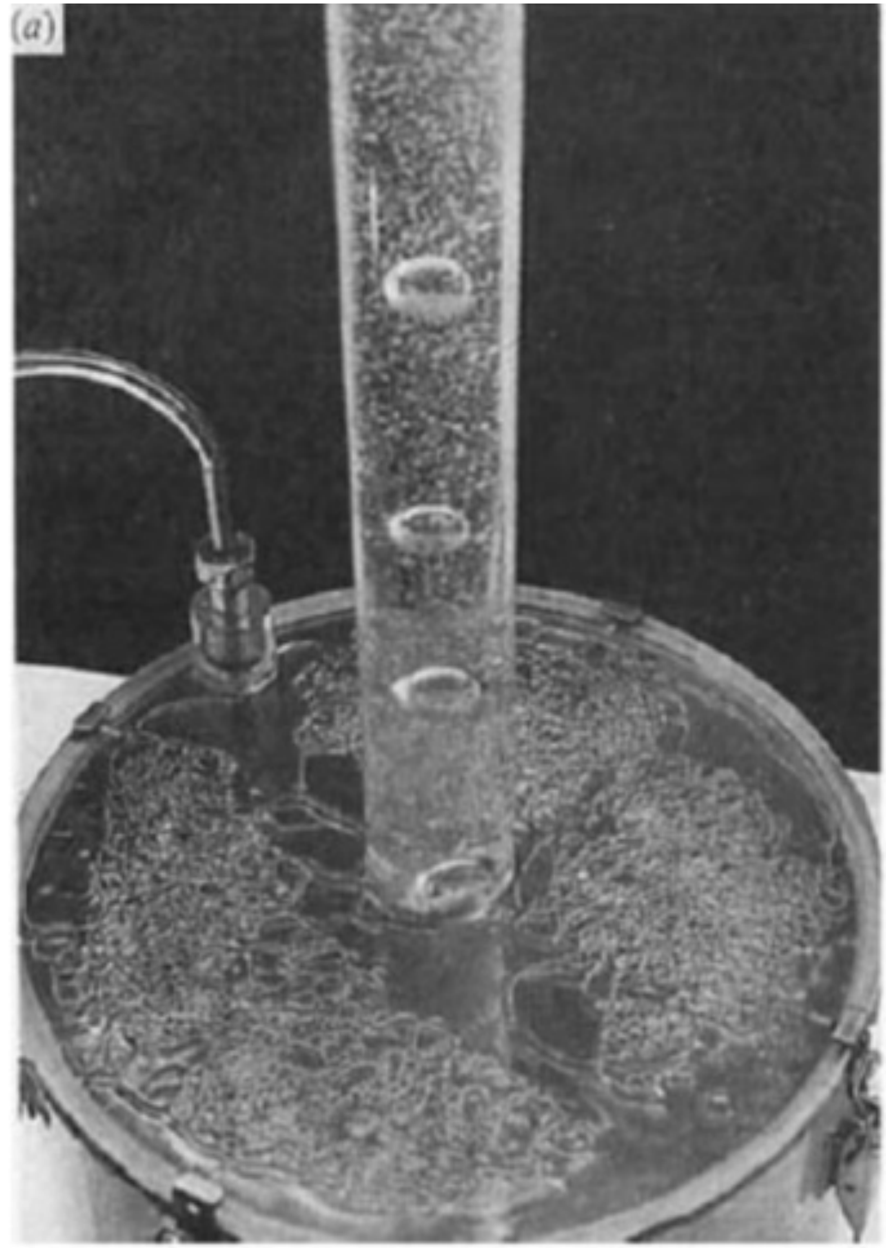


3. Three-phase coupling

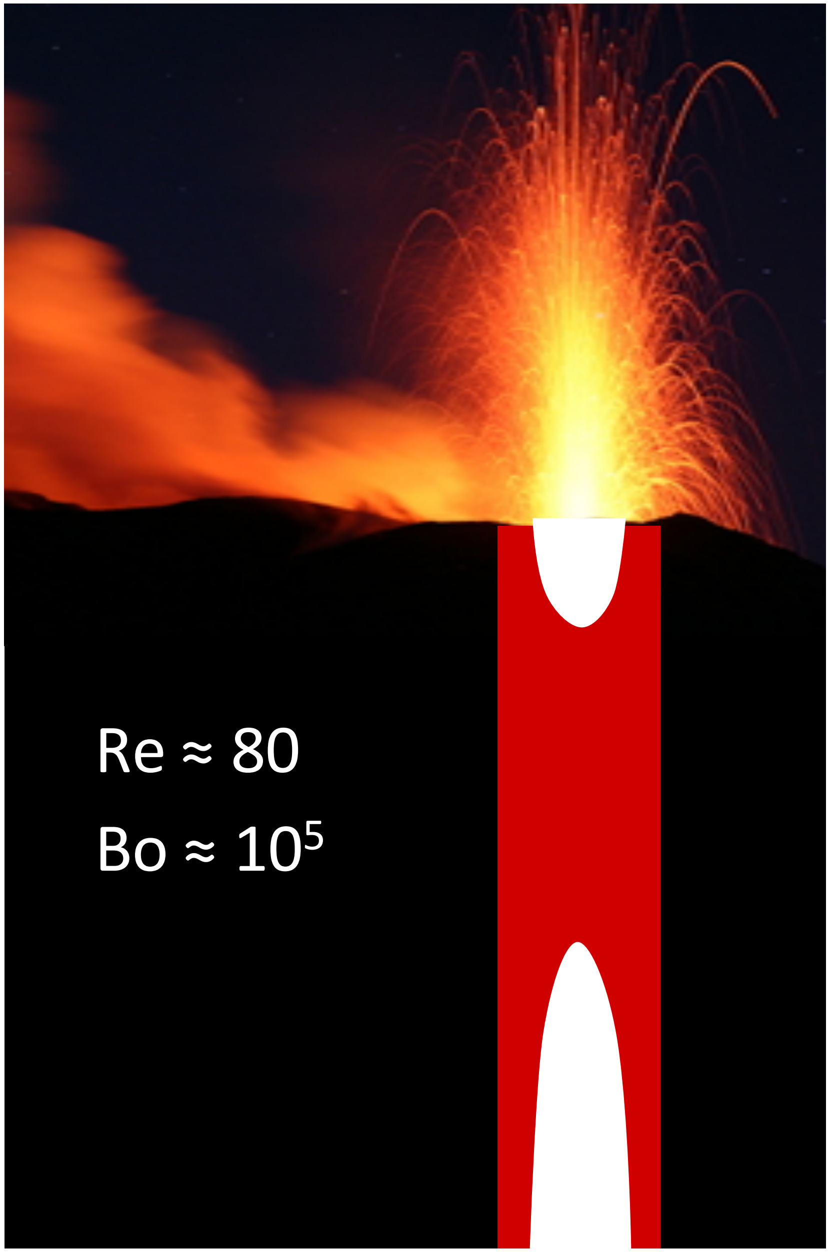


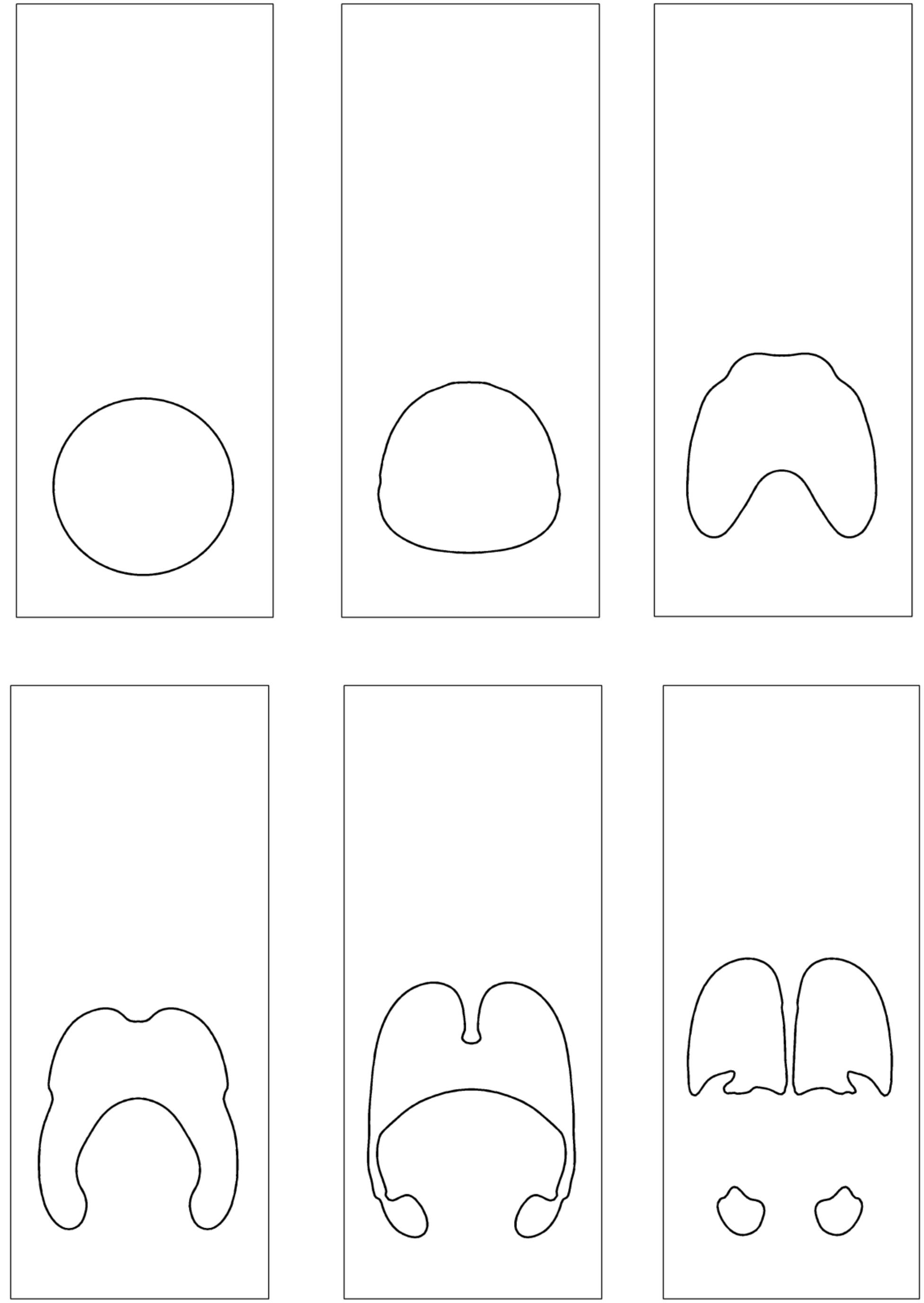
Suckale et al., 2012a; Qin and Suckale, 2017; Taneda 1956; Belien et al., 2010.



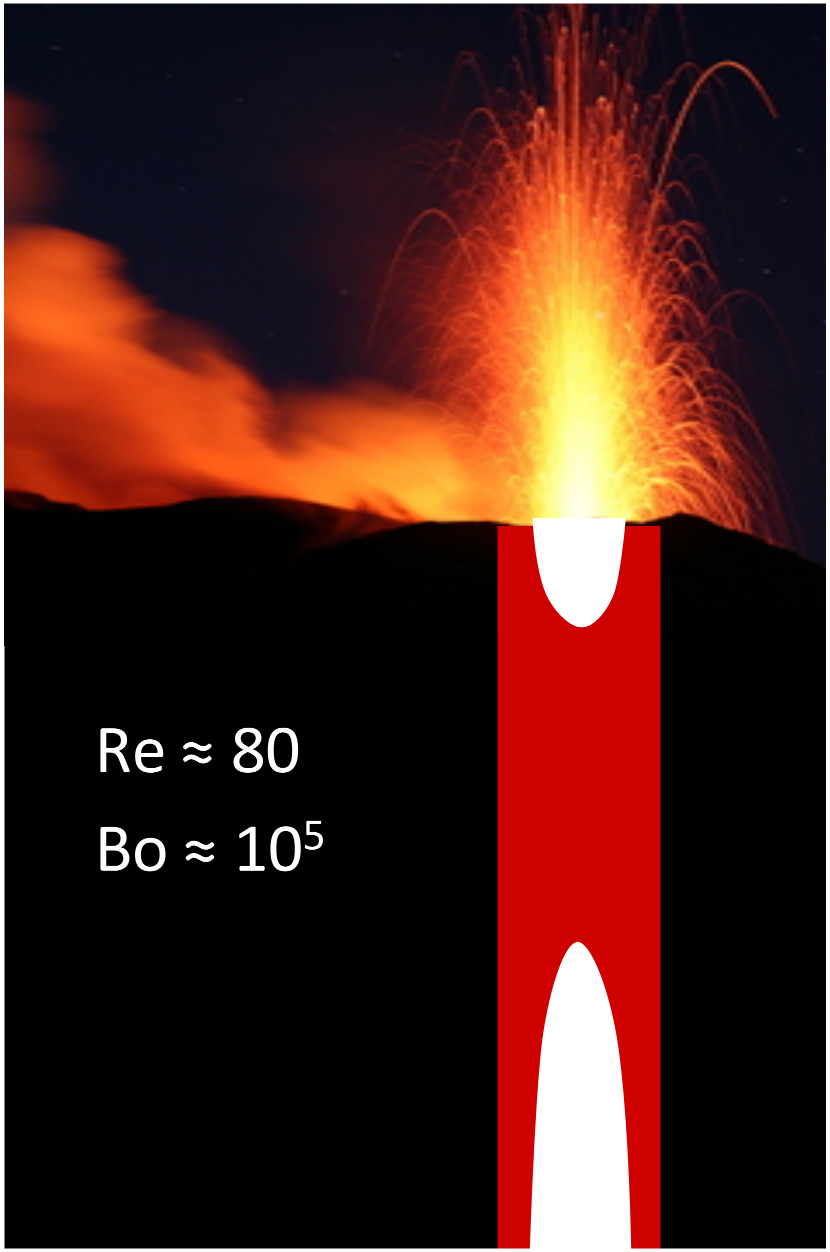


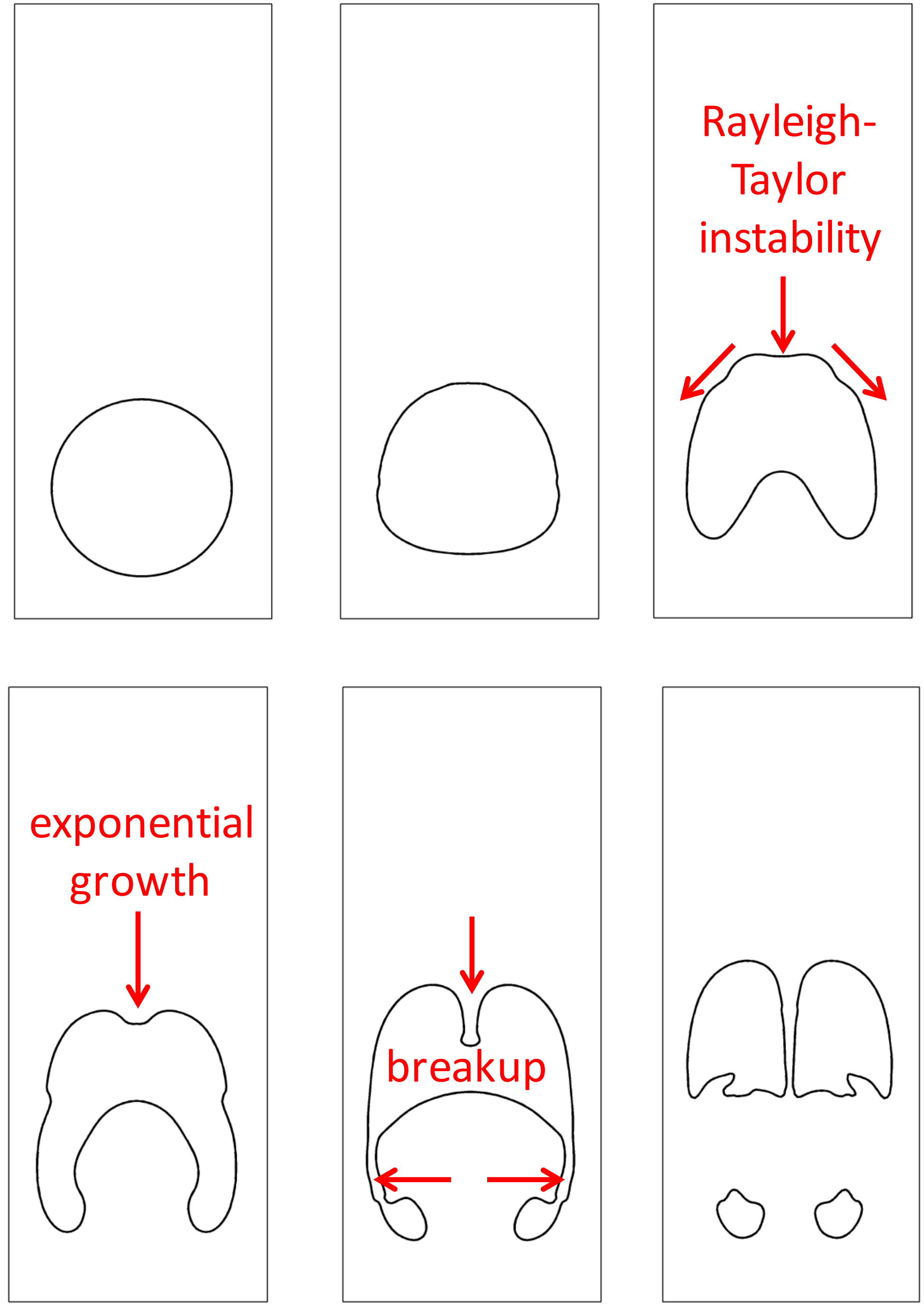
Scaling from Vergniolle et al., 1996; Photo by Jaupart and Vergniolle, 1988;



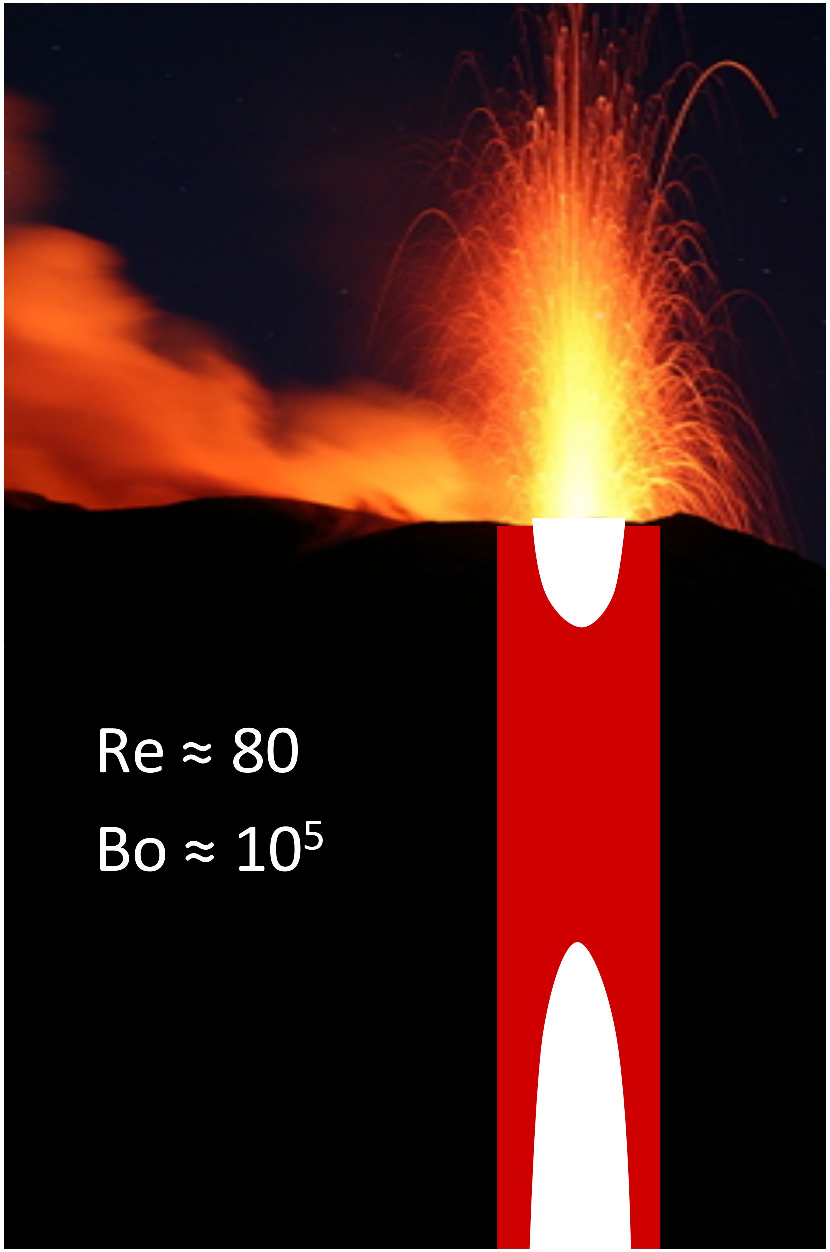


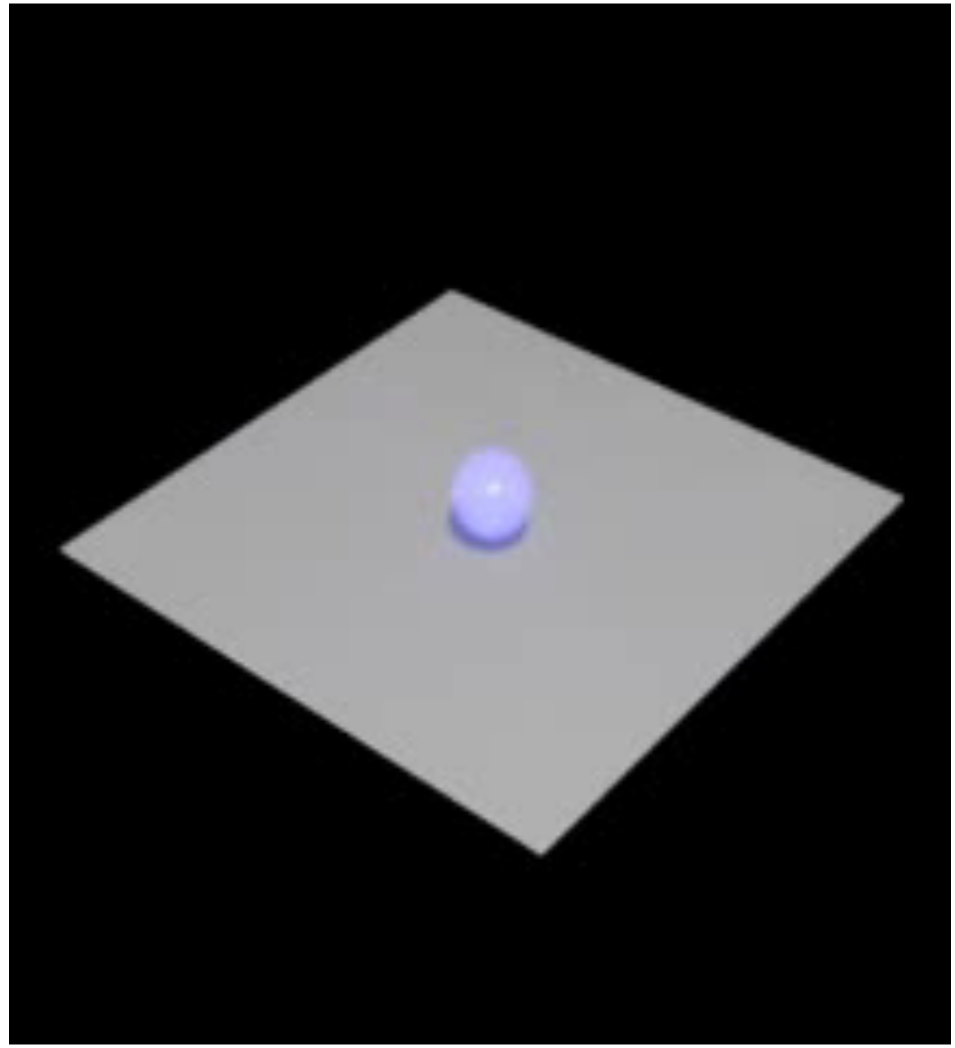
PhD thesis by Jenny Suckale and Suckale et al., 2010a, 2010b





PhD thesis by Jenny Suckale and Suckale et al., 2010a, 2010b





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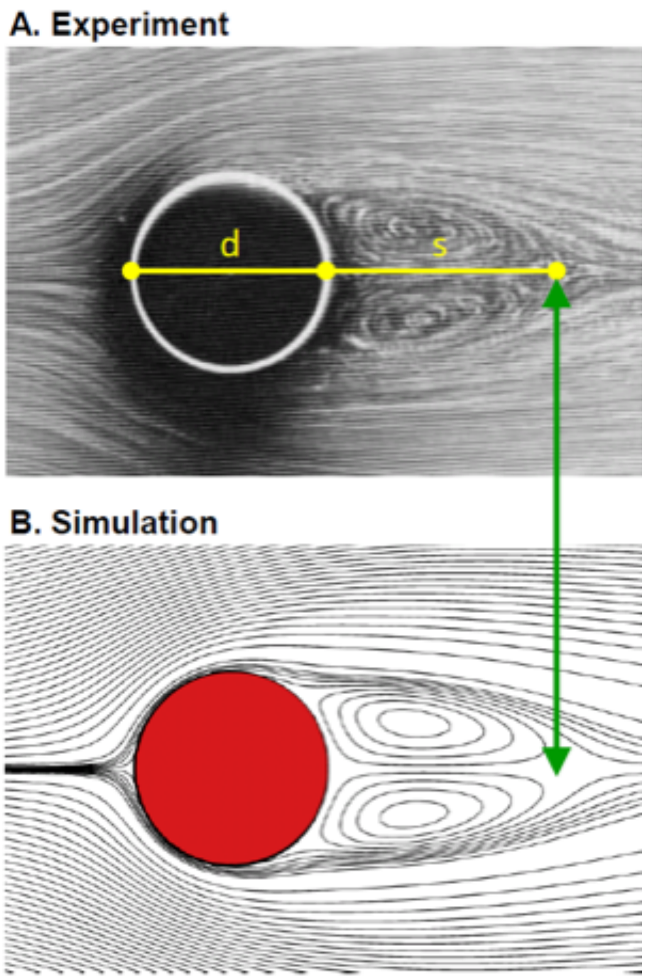
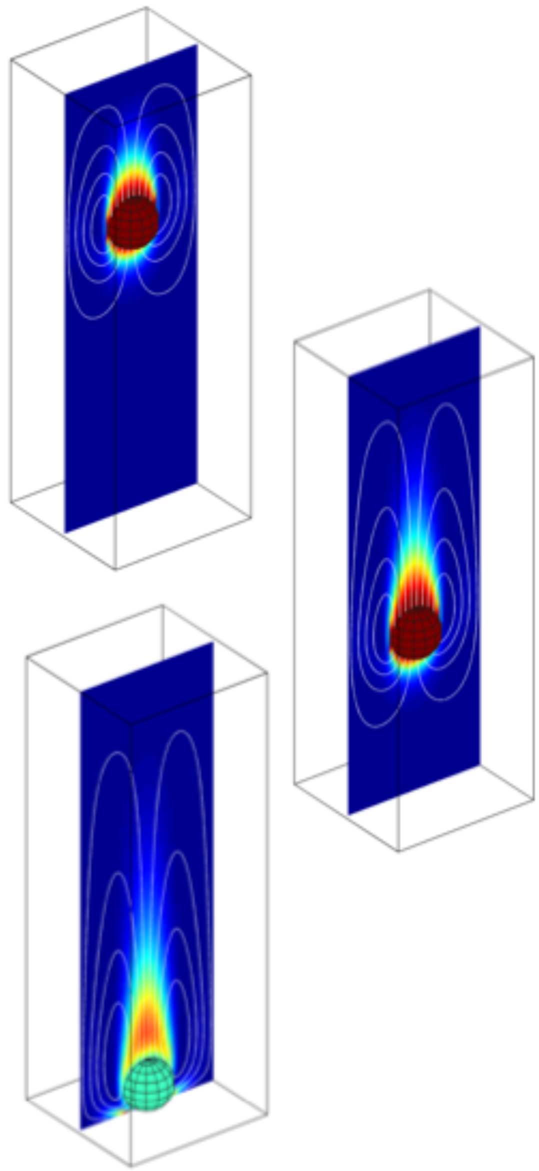
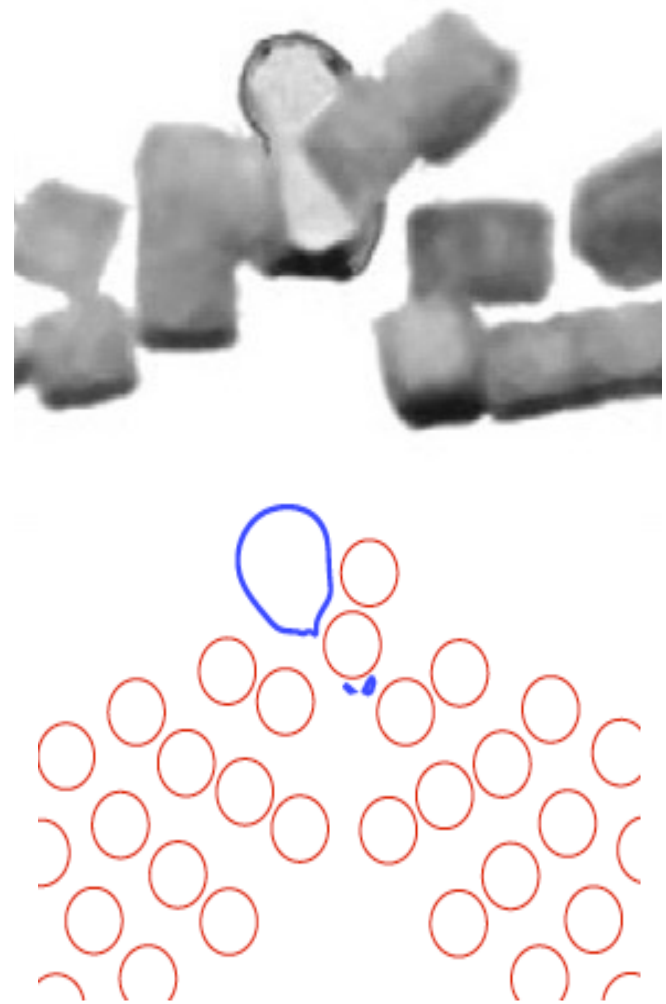


2. Settling speed



3. Three-phase coupling



Taneda 1956 and Belien et al., 2010; Suckale et al., 2012a,b; Qin and Suckale, in review

Governing equations

Tobias Keller

SEGREGATION
conservation of phase momentum

$$\phi^i \Delta \mathbf{v}^{i*} = -\frac{q_\phi^i}{\eta_\phi^i}\left(\phi^i \nabla P^* + \nabla \phi^i \Delta P^{i*} - \nabla \cdot \phi^i \eta_\phi^i \mathbf{D}\left(\mathbf{v}^i\right) - \phi^i \rho^i \mathbf{g}\right)$$

COMPACTION
conservation of phase mass

$$\phi^i \Delta P^{i*} = -\frac{\eta_\phi^i}{r_\phi^i}\left(\phi^i \nabla \cdot \mathbf{v}^* + \nabla \cdot \phi^i \Delta \mathbf{v}^{i*} - \frac{\phi^i}{\rho^i}\frac{D^i \rho^i}{Dt} - \frac{\Gamma_\rho^i}{\rho^i}\right)$$

PHASE EVOLUTION
conservation of phase volume

$$\frac{D^i \phi^i}{Dt} = \nabla \cdot \kappa_\phi^i \nabla \Delta \phi^{i*} - \phi^i \nabla \cdot \mathbf{v}^i - \frac{\phi^i}{\rho^i}\frac{D^i \rho^i}{Dt} - \frac{\Gamma_\rho^i}{\rho^i}$$

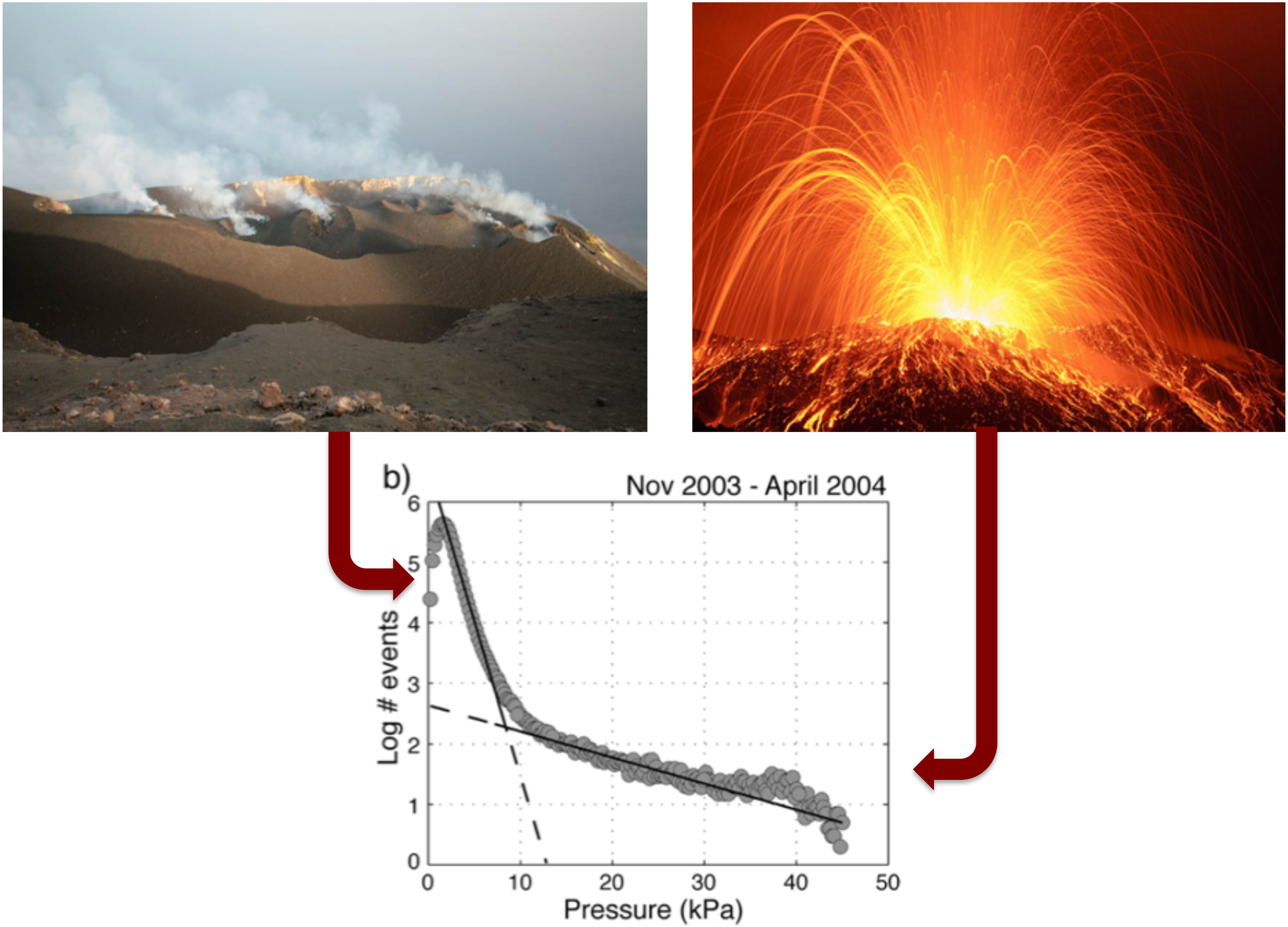
CHEMICAL EVOLUTION
conservation of component mass

$$\frac{D^i c_j^i}{Dt} = \nabla \cdot \kappa_j^i \nabla \Delta c_{j*}^i + \frac{\Delta c_j^{i\Gamma} \Gamma_\rho^i}{\phi^i \rho^i}$$

THERMAL EVOLUTION
conservation of phase energy

$$\frac{D^i T^i}{Dt} = \nabla \cdot \kappa_T^i \nabla T^i - \frac{\kappa_T^i}{d^{i2}} \Delta T^{i*} + \frac{L^{i*} \Gamma_\rho^i}{\phi^i \rho^i c_P^i} + \frac{\Psi^i}{\phi^i \rho^i c_P^i} + \frac{\alpha^i T^i}{\rho^i c_P^i}\frac{D^i P^i}{Dt} + \frac{H^i}{c_P^i}$$

Keller and Suckale, in review



Modified figure 11 from Ripepe et al. 2007